FURTHER MATHS TRANSITION WORK

WATCH……

3Blue1Brown, by Grant Sanderson, is some combination of maths and entertainment, depending on your disposition. The goal is to use animations to explain difficult problems and simplify them with changes in perspective

https://www.youtube.com/c/3blue1brown

Eddie Woo's videos are a great source of revision and to make sure you fully understand the content

https://www.youtube.com/c/misterwootube

Other YouTube channels include; Numberphile, BlackpenRedpen, Michael Penn, Dr Peyam and Tomrocksmaths

READ…

Plus magazine is full of articles on a range of and interesting and advanced areas of mathematics.

https://plus.maths.org/content/

If you're looking to go beyond the school syllabus and see what maths is really like, have a look through Cambridge's reading list to get some inspiration.

https://www.maths.cam.ac.uk/undergrad/admissions/files/reading-list.pdf

COURSE DETAILS…

EXAM BOARD – EDEXCEL

The specification can be found at the link below:

https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/as-l3-further-mathematics-specification.pdf

GET AHEAD……

The first few topics we will be covering are:

Complex Numbers, Argand diagrams, Series, Roots of Polynomials

Either purchase your own textbook or possibly borrow one from school to make a head start in reading up these topics.

GET ORGANISED….

You need to have: Checklists, notes (summaries and detailed), knowledge organisers, practice questions, homework, assessments and DIRT.

Paper based, or virtual exercise books must be kept-up to date each week; you are expected to bring these with you to every lesson.

All records can be kept either electronically or in paper format and must be readily accessible as they may be called upon for inspection/audit by KSA staff at any time.

FURTHER MATHS TRANSITION WORK

TASK……

You will have received the task below as part of you Pure Maths induction. In so far as the further maths summer work is concerned you are expected to read through and complete the questions in the attached three worksheets submitting your solutions one per week during the first three weeks of autumn term.

https://vle.woodhouse.ac.uk/topicdocs/maths/pdf/transition_work.pdf

Answers are included but your solutions must be in full and make honest corrections where you have made mistakes to help identify what you need to work on.

QUESTIONS, QUERIES AND COMMENTS…..

Use this section to make a note of anything you would like to ask your teacher about when the course starts in September.

A LEVEL FURTHER MATHS BRIDGING WORK – WEEK 1

Algebra

- Be confident using the factor & remainder theorem, including to factorise a polynomial up to order 3; solve a cubic equation by factorisation & long division
- Understand and be able to apply the binomial expansion of $(a + b)^n$ where n is a positive integer
- Calculate probabilities using the binomial distribution
- Know and use the formula for the gradient of a line, distance between two points, mid-point of a line segment & equation of a straight line

Be confident using the factor & remainder theorem, including to factorise a polynomial up to order 3; solve a cubic equation by factorisation & long division

Example 1: Solve the equation $f(x) = x^3 - 4x^2 + x + 6 = 0$.

Remember that each root is a factor of 6.
By trial: $f(2) = 8 - 16 + 2 + 6 = 0$,
so $(x - 2)$ is a factor of $f(x)$ and $x = 2$ is a root of the equation $f(x) = 0$.
Similarly, $f(3) = 27 - 36 + 3 + 6 = 0$, so $x = 3$ is a root.
Since $2 \times 3 \times -1 = -6$, the third root must be $x = -1$.
The solution is $x = -1$, $x = 2$ or $x = 3$.

Example 2: Find the remainder when $f(x) = x^3 + 2x^2 + 6$ is divided by $(x + 1)$

Substitute in -1 for x in the expression
$(-1)^3 + 2(-1)^2 + 6$
$= -1 + 2 + 6$
$= 7$
$\therefore$ The remainder is 7

Watch this video for extra help: https://youtu.be/Nw9vxiR9wXU

Q1. (i) Show that $(x - 2)$ is a factor of $f(x) = x^3 + x^2 - 4x - 4$.
(ii) Show that $(x + 1)$ is a factor of $f(x) = x^3 + x^2 + x + 1$.

Q2. Find the remainder when $f(x) = x^3 - 2x^2 + x + 4$ is divided by
(i) $(x - 2)$
(ii) $(x + 2)$.

Q3. Use the factor theorem to solve the equation $x^3 - 3x^2 - x + 3 = 0$.

Example 3:

The function $f(x)$ is defined by $f(x) = x^3 - 5x^2 + 2x + 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 1)$.

(ii) Solve the equation $f(x) = 0$.

(i) Substitute in -1 for x in the expressio

$(-1)^3 - 5(-1)^2 + 2(-1) + 8$

$= -1 - 5 - 2 + 8$

$= 0$

The remainder is 0

$\therefore$ (x + 1) is a factor of the polynomi

(ii) We know one factor of the poynomial so we can use lo division to find the others

$$
\begin{array}{r|l}
 & x^2 - 6x + 8 \\
\hline
(x+1) & x^3 - 5x^2 + 2x + 8 \\
 & -(x^3 + x^2) \\
 & \quad -6x^2 + 2x \\
 & \quad -(-6x^2 - 6x) \\
 & \qquad 8x + 8 \\
 & \qquad -(8x + 8) \\
 & \qquad 0
\end{array}
$$

$\Rightarrow (x+1)(x^2 - 6x + 8) = 0$

$\Rightarrow (x+1)(x-2)(x-4) = 0$

$\Rightarrow x = -1, 2, 4$

Watch this video:
https://youtu.be/gbKxGxQN56k

Q4.

The function $f(x)$ is defined by $f(x) = x^3 + 2x^2 - 5x - 6$.

(i) Show that when $f(x)$ is divided by $(x - 3)$ the remainder is 24.

(ii) Show that $(x - 2)$ is a factor of $f(x)$.

(iii) Hence solve the equation $f(x) = 0$.

Understand and be able to apply the binomial expansion of $(a + b)^n$ where n is a positive integer

Watch this: https://youtu.be/wGO3OoYDymQ

Example 1:

Find the first four terms of the expansion $(1 - 2x)^8$

$$(1-2x)^8 = 1 + \frac{8}{1}(-2x) + \frac{8\times 7}{1\times 2}(-2x)^2 + \frac{8\times 7\times 6}{1\times 2\times 3}(-2x)^3 + \ldots$$

$$= 1 - 8\times 2x + 28\times 4x^2 - 56\times 8x^3 + \ldots$$

$$= 1 - 16x + 112x^2 - 448x^3 + \ldots$$

The first four terms are $1, -16x, 112x^2$ and $-448x^3$.

Q1. Write down the first four terms in the expansion of $(1 + x)^6$.

Q2. Find the constant term in the expansion of $\left(x - \frac{1}{x}\right)^4$.

Q3. **(i)** Expand $(2 + x)^7$ in ascending powers of x up to and including the term in x^3.

(ii) Use your expansion with an appropriate value of x to find an approximate value of 1.99^7.
Give your answer to 4 decimal places.
Show your working clearly, giving the numerical value of each term.
(Writing down the value of 1.99^7 from your calculator will earn no mark.)

Calculate probabilities using the binomial distribution

Watch this: https://youtu.be/--WfdtF64xA

Example 1:

A spinner has five sides, numbered 1 to 5. When it is spun, each side is equally likely to come to rest on the table. What is the probability that the side with number 5 lands on the table exactly three times in in five spins?

Because each side is equally likely to rest on the table, $p = \frac{1}{5}$ and $q = \frac{4}{5}$.

P(3 successes) is given by the third term in the expansion of $(p+q)^5$.

$\Rightarrow \quad P(3 \text{ successes}) = 10\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2$

$= 10 \times \frac{16}{5^5} = \frac{32}{625} = 0.0512$

Example 2:

A normal die is thrown three times.

What is the probability of getting at least one six?

P(at least one six) = 1 – P(no six)

$= 1 - \left(\frac{1}{6}\right)^3 = 1 - 0.0046 = 0.995$

Q1.

Find the probability of obtaining exactly two sixes when a normal die is thrown five times.

Q2.

In a game, 5 normal dice are rolled.
What is the probability that

(i) no sixes are rolled

(ii) at least 1 six is rolled

(iii) exactly 3 sixes are rolled?

Q3.

A factory has three machines that work at the same time. Each machine works independently of the others and the probability that it will break down on any given day is 0.1. Any machine that breaks down is not used again that day and is mended and serviced overnight. In any given day, what is the probability that at least one machine is working at the end of the day?

Know and use the formula for the gradient of a line, distance between two points, mid point of a line segment & equation of a straight line

Watch this: https://youtu.be/MS9EWKWhAjo

Watch this: https://youtu.be/YZmKp9Uv8ic

Example 1:

The vertices of a quadrilateral are A (–2, 0), B (2, 2), C (7, –3) and D (0, –4).

(i) Calculate the gradients of the diagonals AC and BD and state a geometrical fact about these lines.

(ii) Show that the midpoint of BD lies on AC.

(iii) Find the equation of the line AC.

Example 2:

Find the distance between the points (–1, –3) and (4, 2).

Q1. Write down the gradients of the following lines.

(i) $y = 2x + 1$ **(ii)** $2y = x + 1$ **(iii)** $3y + 2x = 4$

Q2. You are given the co-ordinates of the four points A (1, 2), B (4, 6), C (8, 3) and D (5, –1).
Show that ABCD is a rectangle.

Q3. (i) Find the equation of the line parallel to $x + 2y = 3$ that passes through the point $(4, 6)$.

(ii) Find the equation of the line perpendicular to $x + 2y = 3$ that passes through the point $(4, 6)$.

Q4. Points A and B have co-ordinates $(1, 3)$ and $(7, 5)$.

(i) Find the midpoint of AB.

(ii) Find the distance between the points A and B.

Q5. Find the equations of the following lines.

(i) Through the points $(-1, -3)$ and $(4, 2)$.

(ii) Through $(4, 3)$ with gradient -3.

Q6. (i) Show that the two lines whose equations are given below are parallel.

$$y = 4 - 2x \qquad\qquad 4x + 2y = 5$$

(ii) Find the equation of the line which is perpendicular to these two lines and which passes through the point $(1, 6)$.

CHALLENGE PROBLEMS FOR THE WEEK:

NO CALCULATORS ALLOWED :)

Find four prime numbers less than 100 which are factors of $3^{32} - 2^{32}$.

Find the value of

$$\frac{1^4 + 2007^4 + 2008^4}{1^2 + 2007^2 + 2008^2}.$$

A LEVEL FURTHER MATHS BRIDGING WORK – WEEK 2

Coordinate Geometry

- Know that the equation of a circle is $(x-a)^2 + (y-b)^2 = r^2$ is the equation of a circle: centre (a, b) and radius r; find the equation of a circle given key information and convert between this form and $x^2 + y^2 + 2ax + 2by + c = 0$ by multiplying out or completing the squares

Trigonometry

- Apply trigonometry to right angled triangles (Pythagoras & SOHCAHTOA) and non right angled triangles (sine and cosine rules)
- Know and be able to use the identities $\tan\theta = \sin\theta / \cos\theta$ & $\sin^2\theta + \cos^2\theta = 1$ to solve trigonometric equations in given intervals

Equation of a circle is $(x-a)^2 + (y-b)^2 = r^2$ is the eqn of a circle: centre (a, b) and radius r;

Watch this: https://youtu.be/sPJJZV43v14

Example 1:

Find the equation of a circle with centre at the origin and radius 4.

$x^2 + y^2 = r^2$ with $r = 4 \quad \Rightarrow \quad x^2 + y^2 = 16$

Example 2:

Find the equation of the circle with centre (2, 3) and radius 5.

$(x-a)^2 + (y-b)^2 = r^2$ with $r = 5$ and $(a, b) = (2, 3)$

$\Rightarrow \quad (x-2)^2 + (y-3)^2 = 25$

Example 3:

Find the centre and radius of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$.

$$x^2 + y^2 - 2x + 4y - 4 = 0$$
$$\Rightarrow \quad x^2 - 2x + y^2 + 4y - 4 = 0$$
$$\Rightarrow \quad (x^2 - 2x + 1) + (y^2 + 4y + 4) - 4 = 1 + 4 = 5$$
$$\Rightarrow \quad (x-1)^2 + (y+2)^2 = 9$$

This represents a circle with a centre at (1, −2) and radius $\sqrt{9} = 3$.

Q1. Find the equation of a circle with centre at the origin and radius 8. (4, -5)

Q2. Find the equation of the circle with centre and radius 9.

Q3. A circle has equation $x^2 + y^2 - 4x - 6y + 3 = 0$. Find the coordinates of the centre and the radius of the circle.

Q4. (i) A circle has equation $x^2 + y^2 - 2x - 4y - 20 = 0$. Find the co-ordinates of its centre, C, and its radius.

(ii) Find the co-ordinates of the points A and B, where the line $y = x + 2$ cuts the circle.

(iii) Find the angle ACB.

Q5. A (1, 10), B (8, 9) and C (7, 2) are three points.

(i) Find the coordinates of the midpoint, M, of AC.

(ii) Find the equation of the circle with AC as diameter.

(iii) Show that B lies on this circle.

(iv) Prove that AM and BM are perpendicular.

(v) BD is a diameter of this circle. Find the coordinates of D.

Apply trigonometry to right angled triangles (Pythagoras & SOHCAHTOA) and non right angled triangles (Sine and Cosine rules

Watch this: https://youtu.be/jaKLyqiP1EE

Example 1:

In a triangle ABC, $a = 5$, $A = 56°$ and $B = 47°$. Find b.

Using the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$,

$$\frac{5}{\sin 56°} = \frac{b}{\sin 47°}$$

$$\Rightarrow \quad b = \frac{5 \sin 47°}{\sin 56°} = 4.41 \text{ units.}$$

Example 2:

The three sides of a triangle are 6, 7 and 8. Find the size of the largest angle. The largest angle is opposite the longest side, which is 8.

$$\text{Therefore } \cos A = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$$

$$= 0.25$$

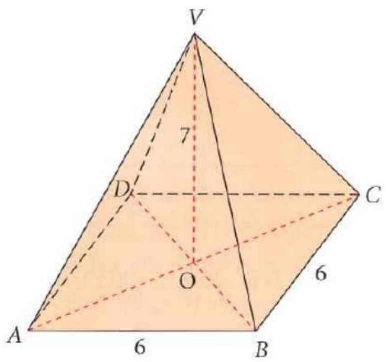
$$\Rightarrow \quad A = 75.5°$$

Q1. In the triangle ABC, $b = 6$, $c = 8$ and $A = 47°$. Find the side a. Find also the area of the triangle.

Q2. In the triangle PQR, $P = 36°$, $QR = 4$ and $PR = 3.5$. Find the angle Q.

Q3. In the triangle ABC, $a = 6$, $b = 8$ and $c = 11$. Find the sizes of the three angles.

Q4. A pyramid $ABCDV$ has a square, horizontal base $ABCD$ of side 6 cm. The vertex V is vertically above the centre of the base O. The pyramid has height 7 cm.



Find the angle that the sloping edge VA makes with the horizontal.

Know and be able to use the identity that $\tan\theta = \sin\theta / \cos\theta$ & $\sin^2\theta + \cos^2\theta = 1$ to solve trigonometric equations in given intervals

Example 1: Find the two values of θ that satisfy $\tan\theta = -0.3$ in the range $0° \leqslant \theta \leqslant 360°$.

$\tan\theta = -0.3$

Using $\tan^{-1}$ on our calculator...

$\theta = -16.7°$.

The period of $\tan\theta$ is $180°$, so we also need

$-16.7° + 180° = 163.3°$.

$163.3° + 180° = 343.3°$

Example 2:

Find the four values of θ that satisfy $\sin 2\theta = 0.4$ in the range $0° \leqslant \theta \leqslant 360°$.

Using $\sin^{-1}$ on our calculator...

$2\theta = 23.6°$.

$2\theta = 180° - 23.6° = 156.4°$.

The period of $\sin x$ is $360°$, so we also need $2\theta = 23.6° + 360° = 383.6°$ and $2\theta = 156.4° + 360° = 516.4°$.

So the four values of θ are $11.8°$, $78.2°$, $191.8°$ and $258.2°$.

Watch this: https://youtu.be/-iC6FSYq4N4

Q1. Solve the equation $\sin\theta = 0.7$ in the range $0° \leqslant \theta \leqslant 360°$.

Q2. Solve the equation $\tan\theta = -1.2$ in the range $0° \leqslant \theta \leqslant 360°$.

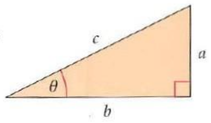
Q3. Solve the equation $\cos 2\theta = 0.7$ in the range $0° \leqslant \theta \leqslant 360°$.

Q4. Find the four values of x in the range $0° \leqslant x \leqslant 360°$ that satisfy the equation $\sin 2x = 0.5$.

Example 3:

(i) Using Pythagoras' theorem on the triangle shown, show that $\sin^2\theta + \cos^2\theta = 1$.

(ii) Hence, given that $\sin\theta = \frac{3}{7}$, find the exact value of $\tan\theta$.



(i) $a^2 + b^2 = c^2$

$\Rightarrow \quad \dfrac{a^2}{c^2} + \dfrac{b^2}{c^2} = \dfrac{c^2}{c^2}$

$\Rightarrow \quad \left(\dfrac{a}{c}\right)^2 + \left(\dfrac{b}{c}\right)^2 = 1$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta = 1$

(ii) $\sin\theta = \frac{3}{7} \quad \Rightarrow \quad \sin^2\theta = \frac{9}{49}$

$\Rightarrow \quad \cos^2\theta = 1 - \frac{9}{49} = \frac{40}{49}$

$\Rightarrow \quad \cos\theta = \sqrt{\frac{40}{49}} = \frac{1}{7}\sqrt{40} = \frac{1}{7}\sqrt{4\times10} = \frac{2}{7}\sqrt{10}$

$\Rightarrow \quad \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{3}{7}}{\frac{2}{7}\sqrt{10}} = \dfrac{3}{2\sqrt{10}} = \dfrac{3}{2\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{3\sqrt{10}}{20}$

Q5. You are given that $\sin\theta = \frac{2}{3}$. Find the exact value of $\cos\theta$.

Q6.

(i) Using the identity $\cos^2\theta + \sin^2\theta = 1$, show that the equation

$$2\cos^2\theta + \sin\theta = 2$$

can be written as

$$2\sin^2\theta - \sin\theta = 0.$$

(ii) Hence find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying the equation

$$2\cos^2\theta + \sin\theta = 2.$$

Example 4:

Show that the equation $2\cos^2\theta = 3\sin\theta$ can be written $2\sin^2\theta + 3\sin\theta - 2 = 0$.
Hence solve the equation $2\cos^2\theta = 3\sin\theta$ in the range $0° \leqslant \theta \leqslant 360°$.

$2\cos^2\theta = 3\sin\theta$

$\Rightarrow \quad 2(1 - \sin^2\theta) = 3\sin\theta$

$\Rightarrow \quad 2 - 2\sin^2\theta = 3\sin\theta$

$\Rightarrow \quad 2\sin^2\theta + 3\sin\theta - 2 = 0$

$\Rightarrow \quad (\sin\theta + 2)(2\sin\theta - 1) = 0$

$\Rightarrow \quad \sin\theta = -2 \text{ or } \sin\theta = \frac{1}{2}$

The first value is not possible. The second value gives $\theta = \sin^{-1}\left(\frac{1}{2}\right)$.
The solutions are $\theta = 30°$ and $\theta = 150°$.

Q7. Calculate the values of x in the range $0° \leqslant x \leqslant 360°$ for which $\sin x = 3\cos x$.

CHALLENGE PROBLEM FOR THE WEEK:

Each of Paul and Jenny has a whole number of pounds.
He says to her: "If you give me £3, I will have n times as much as you".
She says to him: "If you give me £n, I will have 3 times as much as you".
Given that all these statements are true and that n is a positive integer, what are the possible values for n?

A LEVEL FURTHER MATHS BRIDGING WORK – WEEK 3

Calculus

- Differentiate kx^n where n is a positive integer or 0, and the sum of such functions (being aware that the gradient function measures the rate of change of y with x)
- Know that the gradient of the function is the gradient of the tangent at that point; use this to find the equation of a tangent and normal at any point on a curve
- Use differentiation to find stationary points on acurve and determine their nature; sketch the curve
- Integrate kx^n where n is a positive integer or 0, and the sum of such functions and find the equation of a curve, given its gradient function and one point
- Evaluate definite integrals and find the area between a curve & a line or two curves
- Use differentiation and integration with respect to time to solve simple problems involving variable acceleration

Differentiate kx^n where n is a positive integer or 0, and the sum of such functions

Watch this: https://youtu.be/4-QLj2hum0Q

Example 1:

a) Find the gradient function of the function $y = x^3$.

The gradient function is $\frac{dy}{dx} = 3 \times x^{3-1}$
$= 3x^2$.

b) Find the gradient function of the function $y = x^2 + 3x^4$.

The gradient function is $\frac{dy}{dx} = 2x^{2-1} + 4 \times 3x^{4-1}$
$= 2x + 12x^3$.

Q1. Differentiate the following functions.

(i) $y = x^2 + x - 2$

(ii) $y = 2x^3 + 3x - 5$

Example 2:

Find the gradient of the curve $y = 4x^2 - x^3$ at the point $(2, 8)$.

The gradient is $\frac{dy}{dx} = 8x - 3x^2$.

When $x = 2$, $\frac{dy}{dx} = 16 - 12 = 4$.

Q2. Given that $y = x^3 + 2x - 7$, find $\frac{dy}{dx}$.

Q3. Find the gradient of the curve $x^3 + 2x^2 + 6$ at the point (1, 9)

Know that the gradient function measures the rate of change o y with x; that the gradient of the function is the gradient of the tangent at that point. Use this to find the equation of a tangent and normal at any point on a curv

Watch this: https://youtu.be/W0T8qjjijE0

Example 1:

Find the equation of the tangent to the curve $y = x^3 - 2x + 3$ at the point $(2, 7)$.

$y = x^3 - 2x + 3$

$\Rightarrow \quad \frac{dy}{dx} = 3x^2 - 2$

When $x = 2$, $\frac{dy}{dx} = 3 \times 4 - 2 = 10$.

The equation of the tangent is

$y - y_1 = m(x - x_1)$.

the gradient (m) is 10 at the point $(2, 7)$.

Use $x_1 = 2$ and $y_1 = 7$.

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow \quad y - 7 = 10(x - 2)$$
$$\Rightarrow \quad y = 10x - 13$$

Example 2:

Find the equation of the normal to the curve $y = x^3 - 3x^2 - 6x + 11$ at the point $(3, -7)$.

$y = x^3 - 3x^2 - 6x + 11$

$\Rightarrow \quad \frac{dy}{dx} = 3x^2 - 6x - 6$

When $x = 3$, $\frac{dy}{dx} = 3 \times 9 - 6 \times 3 - 6 = 3$

$$m_1 \times m_2 = -1$$
$$\Rightarrow \quad m_2 = -\frac{1}{m_1} = -\frac{1}{3}$$
$$y - y_1 = m(x - x_1)$$
$$\Rightarrow \quad y + 7 = -\tfrac{1}{3}(x - 3)$$
$$\Rightarrow \quad 3y + 21 = 3 - x$$
$$\Rightarrow \quad 3y + x + 18 = 0$$

Q1. Find the equation of the tangent to the curve $y = x^3 + x^2 - 3x + 1$ at the point $(2, 7)$.

Q2. Find the equation of the normal to the curve $y = x^3 - x^2 + 4x - 2$ at the point $(1, 2)$.

Q3. The curve $y = x^2 + 3x + k$ has a tangent with equation $y = 5x + 5$. Find the value of k.

Q4. Find the point on the curve $y = x^2 - 2x + 7$ where the gradient is 8.

Q5. A curve has equation $y = 4x^3 - 5x^2 + 1$ and passes through the point A(1, 0).

(i) Find the equation of the normal to the curve at A. [5]

(ii) This normal also cuts the curve in two other points, B and C. Show that the x-coordinates of the three points where the normal cuts the curve are given by the equation $8x^3 - 10x^2 + x + 1 = 0$. [2]

(iii) Show that the point B $\left(\frac{1}{2}, \frac{1}{4}\right)$ satisfies the normal and the curve. [2]

(iv) Find the coordinates of C. [3]

Use differentiation to find stationary points on a curve and determine their nature

Example 1:

Investigate the nature of the turning point of the curve $y = 2x^3 - 3x^2 - 12x + 24$ that is in the positive quadrant.

$y = 2x^3 - 3x^2 - 12x + 24$

$\Rightarrow \quad \frac{dy}{dx} = 6x^2 - 6x - 12$

When $\frac{dy}{dx} = 0$

$6x^2 - 6x - 12 = 0$

$\Rightarrow \quad x^2 - x - 2 = 0$

$\Rightarrow \quad (x-2)(x+1) = 0$

So the turning point in the positive quadrant is when $x = 2$.

$x = 2 \quad \Rightarrow \quad y = 4$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

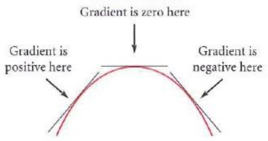
When $x = 1.9$, $\frac{dy}{dx} = -1.74$.

When $x = 2$, $\frac{dy}{dx} = 0$.

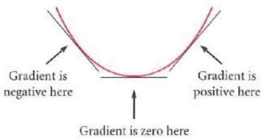
When $x = 2.1$, $\frac{dy}{dx} = 1.86$.

Since the gradient goes from negative to 0 to positive, the turning point is a minimum.

Maximum



Minimum



Watch this: https://youtu.be/MNpaP2eHI4s

Q1. (i) Show that there is a stationary point at $(1, 9)$ on the curve $y = x^3 - 6x^2 + 9x + 5$ and determine the nature of this stationary point.

(ii) Find the co-ordinates of the other stationary point and hence sketch the curve.

Q2. (i) Use calculus to find the stationary points on the curve $y = x^3 - 3x + 1$, identifying which is a maximum and which is a minimum. [6]

(ii) Sketch the curve. [1]

Q3. Find the co-ordinates and the nature of the turning points of $y = x^3 - 4x^2 + 5x - 2$. Sketch the curve.

Q4. Show that the curve $y = x^3 + x^2 + x + 1$ has no turning point.

Q5. The perimeter of a rectangular enclosure is 100 m.

(i) If one side of the enclosure is x m then show that the area, $A\ \text{m}^2$, of the enclosure is given by $A = x(50 - x)$.

(ii) Find the value of x that will give the maximum area.

Integrate kx^n where n is a positive integer or 0, and the sum of such functions

Watch this: https://youtu.be/gMXiOn1auhs

Example 1:

Integrate $6x^3$.

$6 \times \frac{x^4}{4} + c = \frac{3x^4}{2} + c$

Q1. Integrate each of the following.

(i) $5x^3 + 3x^5$

(ii) $x(2x-1)$

Example 2:

Find $\int (6x^2 - 4x)\,dx$.

$$\int (6x^2 - 4x)\,dx = 6 \times \frac{x^3}{3} - 4 \times \frac{x^2}{2} + c$$
$$= 2x^3 - 2x^2 + c$$

Q2. If $\frac{dy}{dx} = 4x^2 + x^3 - 5x^4$, find y.

Q3. You are given that $\frac{dy}{dx} = x^2 + 2x - 3$. Find y.

Q4. Find each of the following.

(i) $\int (3x^2 - 4x + 1)\,dx$

(ii) $\int x(2x-1)\,dx$

(iii) $\int (x+1)(2x-3)\,dx$

Find the equation of a curve, given its gradient function and one point

Example 3:

The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 2x + 1$

Find the equation of the curve, given that it passes through the point (1, 2).

$\frac{dy}{dx} = 3x^2 - 2x + 1 \quad \Rightarrow \quad y = x^3 - x^2 + x + c$

This equation is satisfied by (1, 2), so substitute $x = 1$, $y = 2$.

$\Rightarrow \quad 2 = 1 - 1 + 1 + c$

$\Rightarrow \quad c = 1$

The equation is $y = x^3 - x^2 + x + 1$.

Q5. The gradient function of a curve is given by $\frac{dy}{dx} = 2x - 5$. The curve passes through the point (1, 2). Find the equation of the curve.

Q6. The gradient function of a curve is given by $\frac{dy}{dx} = 2 + 2x - x^2$. Find the equation of the curve given that it passes through the point (3, 10).

Evaluate definite integrals and find the area between a curve & a line or two curves

Watch this: https://youtu.be/hKi8krTg380

Watch this: https://youtu.be/3JeIt_N1LrU

Example 1:

Evaluate $\int_2^3 (6x^2 - 4x)\,dx$

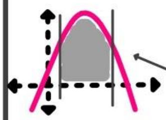
$= \left[2x^3 - 2x^2\right]_2^3$

$= \left(2\times 3^3 - 2\times 3^2\right) - \left(2\times 2^3 - 2\times 2^2\right)$

$= (54 - 18) - (16 - 8)$

$= 36 - 8 = 28$

Example 2: Find the area between the curve $y = 3 + 4x - x^2$, the x-axis and the lines $x = 1$ and $x = 4$.



Sketch first!

Area $= \int_1^4 (3 + 4x - x^2)\,dx$

$= \left[3x + 2x^2 - \frac{x^3}{3}\right]_1^4$

$= \left(3\times 4 + 2\times 4^2 - \frac{4^3}{3}\right) - \left(3 + 2 - \frac{1}{3}\right)$

$= \left(44 - \frac{64}{3}\right) - \left(5 - \frac{1}{3}\right)$

$= 39 - \frac{63}{3}$

$= 39 - 21 = 18$ units^2

Q1. Evaluate

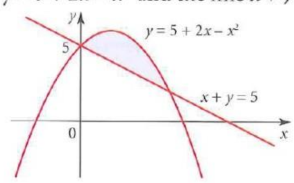
$\int_0^8 (8x - x^2)\,dx$

Q2. Find the area enclosed by the curve $y = 3 + 2x - x^2$ and the x-axis.

Q3. Find the area between the curve $y = 5 + 4x - x^2$, the x-axis and the lines $x = 2$ and $x = 4$

Example 3:

Find the area enclosed by the curve $y = 5 + 2x - x^2$ and the line $x + y = 5$.



Substitute $y = 5 - x$ to find the intersections with the curve.

$5 - x = 5 + 2x - x^2$

$\Rightarrow \quad x^2 - 3x = 0$

$\Rightarrow \quad x = 0$ or $x = 3$

Area $= \int_0^3 ((5 + 2x - x^2) - (5 - x))\,dx$

$= \int_0^3 (3x - x^2)\,dx$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \left(\frac{27}{2} - \frac{27}{3}\right) - 0 = \frac{27}{6} = \frac{9}{2}$ units^2

Q4. Show that the line $x + y = 4$ cuts the curve $y = 7 - 5x + x^2$ at A and B, where the co-ordinates of A and B are (1, 3) and (3, 1) respectively. Find the area enclosed.

Q5. Find the area between the curve $y = 13 - 6x + x^2$ and the curve $y = 3 + 6x - x^2$.

Q6. Find the area enclosed between the curves $y = 2 + 4x - x^2$ and $y = 10 - 6x + x^2$.

Use differentiation and integration with respect to time to solve simple problems involving variable acceleration

Example 1:

A body moves along a straight line. As it passes a point O it is travelling at 4 ms^{-1}. The acceleration t seconds after passing O given by $a = 2 + t$.

Find the velocity after 5 seconds and the displacement at that time.

$$a = 2 + t \quad \Rightarrow \quad v = 2t + \frac{t^2}{2} + c$$

When $t = 0, v = 4 \quad \Rightarrow \quad c = 4$

$$\Rightarrow \quad v = 2t + \frac{t^2}{2} + 4$$

Displacement is found by integrating.

$$v = 2t + \frac{t^2}{2} + 4 \quad \Rightarrow \quad s = t^2 + \frac{t^3}{6} + 4t + c$$

When $t = 0, s = 0 \quad \Rightarrow \quad c = 0$

$$\Rightarrow \quad s = t^2 + \frac{t^3}{6} + 4t$$

When $t = 5, v = 10 + \frac{25}{2} + 4 = 26\frac{1}{2}$

$$\Rightarrow \quad s = 25 + \frac{125}{6} + 20 = 65\frac{5}{6}$$

The velocity is 26.5 ms^{-1} and the displacement is 65.8 m.

Watch this: https://youtu.be/W8UBPiXiZ1M

Q1.

A particle moves in a straight line with a constant acceleration of 2 ms^{-2}. At a point O it has a velocity of 4 ms^{-1}. Find its displacement and velocity 4 seconds after passing O.

Q2.

A body moves along a straight line with acceleration given by $a = 2 - t$. As it passes a point O it is travelling with velocity 8 ms^{-1}. Find the velocity and the displacement after 6 seconds.

Remember:

Displacement

$\frac{dy}{dx}$ ↓ ↑ $\int$

Velocity

$\frac{dy}{dx}$ ↓ ↑ $\int$

Acceleration

Q3. A car travelling at 30 ms^{-1} passes a point A on a motorway. It slows at a constant rate so that 10 seconds later it is travelling at 15 ms^{-1}. Find the deceleration and the distance travelled in this time.

Q4. A particle moves along a straight line. t seconds after passing point O, the displacement s metres is given by $s = 9t^2 - t^3$. Find the time, other than $t = 0$, at which the particle is instantaneously at rest and the displacement at this time. What happens after this time?

CHALLENGE PROBLEM FOR THE WEEK:

Find all positive integers m, n, where n is odd, that satisfy

$$\frac{1}{m} + \frac{4}{n} = \frac{1}{12}.$$

Let n be an integer greater than 6. Prove that if $n - 1$ and $n + 1$ are both prime, then $n^2(n^2 + 16)$ is divisible by 720. Is the converse true?