## Factorising a quadratic expression

| 1. | Factorising a quadratic in the form of $a x^{2}+$ $b x+c$ | Multiply to 5 <br> Factorise $x^{2}+5 x+6-$ Add to 6 <br> 2 and 3 add to 5 <br> 2 and 3 multiply to 6 $(x+2)(x+3)$ <br> Check: $(x+2)(x+3)=x^{2}+5 x+6$ |  |
| :---: | :---: | :---: | :---: |
| 2. | Difference of two squares | A special type of quadratic which only has two terms. |  |
|  |  | One term is subtracted from the other |  |
|  |  | $\begin{aligned} & x^{2}-25=x^{2}-5^{2} \\ & y^{2}-49=y^{2}-7^{2} \\ & a^{2}-16=a^{2}-4^{2} \end{aligned}$ | $\begin{aligned} & =(x+5)(x-5) \\ & =(y+7)(y-7) \\ & =(a+4)(a-4) \end{aligned}$ |
| 3. | Factorising a quadratic in the form of $a x^{2}+$ $b x+c$ where $a>1$ | By inspection $\begin{aligned} & 4 x^{2}+20 x+9 \\ & (4 x+9)(x+1) \\ & (4 x+3)(x+3) \\ & (2 x+9)(2 x+1) \\ & (2 x+3)(2 x+3) \end{aligned}$ | Splitting the middle $\begin{gathered} 4 x^{2}+20 x+9 \\ 4 x^{2}+2 x+18 x+9 \\ 2 \boldsymbol{x}(2 x+1)+\mathbf{9}(2 x+1) \\ (2 x+1)(2 x+9) \end{gathered}$ |

## Solving quadratic equations/functions

| 4. | By factorising | Take you factorised form and set each bracket equal to zero <br> Solve each separate linear equation to find the solutions/roots | $\begin{gathered} x^{2}+4 x+3=0 \\ (x+3)(x+1)=0 \\ x+3=0 \quad x+1=0 \\ \text { So } \quad \text { So } \\ x=-3 \quad n=-1 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| 5. | The quadratic formula | A formula to find the solutions a quadratic equation in the form of $a x^{2}+b x+c$ | $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ |


| 6. | Completing th | quare | $x^{2}+b x+c$ can be writt the form $\left(x+\frac{b}{2}\right)^{2}-\left(\frac{b}{2}\right)^{2}+c$ |  | If $a$ is greater than 1 this will need to be factored out first! |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Simultaneous equations |  |  |  |  |  |
| 7. | Simultaneous equations | Two equations where there are two unknown which have the same value in each |  |  |  |
| Solving simultaneous equations |  |  |  |  |  |
|  |  | Add or subtract one equation from another to eliminate a variable |  |  |  |
| 8. | Elimination | If the matching coeefieicents have the same sign then subtract the equations <br> $\checkmark$ Same <br> $\checkmark$ Subtract <br> $\checkmark$ Substitute |  |  | the matching coefficients have different ns then add the equations <br> $\checkmark$ Different <br> $\checkmark$ Add <br> $\checkmark$ Substitute |
| 9. | Substitution | Rearrange so the subject of one equation is a single variable |  |  |  |
|  |  | Substitute this into the second equation |  |  |  |
| 10. | Graphically | The points of intersection of two graphs are the solutions to the simultaneous equations |  |  |  |

## Inequalities

| 11. | Inequality | The relationship between two expressions that are not equal |  |
| :---: | :---: | :---: | :---: |
| 12. | $=$ | Equal to |  |
| 13. | \# | Not equal to |  |
| 14. | < | Less than | $$ |
| 15. | > | Greater than | $x>5$ |
| 16. | $\leq$ | Less than or equal to | $x \leq 5$ |
| 17. | $\geq$ | Greater than or equal to | $x \geq 3$ |
| 18. | Inclusive | Gives a finites rnage of solutions | e.g. $3<x \leq 8$ |
| 19. | Exclusive | Gives an infinite range of solutions | e.g. $x>5 \quad-4 \leq x$ |
| 20. | Integer | A whole number that can be positive negative or zero |  |
| 21. | Solve | Inequalities are solved in the same way as solving equations |  |
|  |  | Only exception: if you multiply or divide by a negative number you must swap the sign e.g. less than to greater than |  |
| 22. | List integers solutions | Give the integers that satisfy the inequality |  |
|  |  | e.g. $x>6$ integer solutions are $6,7,8 . \ldots$. |  |
|  |  | e.g. $-5<x \leq 5$ integer solutions are $-4,-3,-2,-1,0,1,2,3,4,5$ |  |
| 23. | Represent on a number line | An empty circle shows the value is not included | $\square$ |
|  |  | A shaded circle shows the value is included |  |
|  |  | An arrow shows that the solution continues to infinity |  |

24. | Inequalities |
| :---: |
| On graphs |

| Probability - definitions |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Probability | The extent to which an event is likely to occur | For equally likely outcomes the probability that an event will happen is$P=\frac{\text { number of successful outcomes }}{\text { total number of possible outcomes }}$ |  |
|  |  | Written as a fraction, decimal or percentage |  |  |
| 2. | Theoretical probability | Calculated without doing an experiment |  |  |
| 3. | Experimental probability | Probabilities based on the data collected during an experiment | $\text { estimated probability }=\frac{\text { frequency of event }}{\text { total frequency }}$ |  |
|  |  | Also known as estimated probability |  |  |
|  |  | The more trials you do the more reliable your set of results |  |  |
| 4. | P() notation | P () mean $s$ the probability of the thing inside the brackets happening e.g. P (tails) |  |  |
| 5. | Experiment | A repeatable process that gives rise to a number of outcomes |  |  |
| 6. | Relative frequency | In an experiment, how often something happens as a proportion of the number of trials | $\underline{\text { Relative frequency }=\frac{\text { how often something happens }}{\text { all outcomes }}}$ |  |
| 7. | Predictions | You can predict the number of outcomes you will get using relative frequency |  |  |
|  |  | Predicted number of outcomes $=$ probability $\times$ number of trials |  |  |
| 8. | Event | A collection of one or more outcomes |  |  |
| 9. | Independent | When one event has no effect on another | Here $P(A$ and $B)=P(A) \times P(B)$ |  |
| 10. | Dependent | When the outcome of one event, changes the probability of the next event |  |  |
| 11. | Exhaustive | Events are exhaustive if they cover all possible outcomes |  |  |
| 12. | Biased | Unfair |  |  |
| 13. | Unbiased | Fair |  |  |
| 14. | Sample space | The set of all possible outcomes |  |  |
| 15. | Sample space diagram | A diagram showing all possible outcomes from an experiment |  |  |
| 16. | Venn diagram | Can be used to represent events graphically |  |  |




| 12. | Units of length | Metric units of length are millimetres, centimetres, metres and kilometres |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $10 \mathrm{~mm}=1 \mathrm{~cm}$ | $100 \mathrm{~cm}=1 \mathrm{~m}$ | $1000 \mathrm{~m}=1 \mathrm{~km}$ |
| 13. | Units of area | Metric units of length are millimetres ${ }^{2}$, centimetres ${ }^{2}$, metres $^{2}$ and kilometres $^{2}$ |  |  |
|  |  | $1 \mathrm{~cm}^{2}=100 \mathrm{~mm}^{2}$ |  |  |
| 14. | Units of volume | Metric units of length are millimetres ${ }^{3}$, centimetres ${ }^{3}$, metres ${ }^{3}$ and kilometres ${ }^{3}$ |  |  |
|  |  | $1 \mathrm{~cm}^{3}=1000 \mathrm{~mm}^{3}$ |  |  |
|  |  | $1 \mathrm{~m}^{3}=1000000 \mathrm{~cm}^{3}$ |  |  |
| 15. | Units of capacity | Metric units of capacity are millilitres, centilitres and litres |  |  |
|  |  | $10 \mathrm{ml}=1 \mathrm{cl}$ |  | $1000 \mathrm{~m} /=100 \mathrm{c}=1 /$ |
| 16. | Capacity and volume conversions | $1 \mathrm{~cm}^{3}=1 \mathrm{~m} /$ |  | $1000 \mathrm{~cm}^{3}=1 /$ |



| 23. | Reverse percentage | Use when asked to find the priginal amount after a percentage increase or decrease. |  |
| :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Original Value } \times \text { Multiplier }=\text { New Value } \\ & \text { Original Value }=\frac{\text { New Value }}{\text { Multiplier }} \end{aligned}$ |  |
| 24. | Interest | A fee paid for borrowing money or money earnt through investing. |  |
| 25. | Simple interest | Interest that is calculated as a percentage of the original | $\begin{aligned} & \quad \text { I = Prt } \\ & \text { I - Interest } \\ & \text { P - Original amount } \\ & \text { r - interest rate } \\ & t \text { - time } \\ & \hline \end{aligned}$ |
| 26. | Compound interest | When interest is calculate on the original amount and any previous interest <br> Or $\quad$ Original $\times$ Multiplier ${ }^{\text {time }}$ | $\begin{aligned} & \qquad \boldsymbol{P}\left(\mathbf{1}+\frac{\boldsymbol{R}}{\mathbf{1 0 0}}\right)^{n} \\ & \mathrm{P} \text { - Original amount } \\ & \mathrm{R} \text { - Interest rate } \\ & \mathrm{n} \text { - the number of interest periods (e.g. yrs) } \end{aligned}$ |
| 27. | Tax | A financial charge placed on sales or savings by the government e.g. VAT |  |
| 28. | Loss | Income minus all expenses, resulting in a negative value |  |
| 29. | Profit | Income minus all expenses, resulting in a positive value |  |
| 30. | Depreciation | A reduction in the value of a product over time |  |
| 31. | Annual | Means yearly |  |
| 32. | Per annum | Means per year |  |
| 33. | Salary | A fixed regular payment, often paid monthly |  |


| Pro | tion graphs |  |  |
| :---: | :---: | :---: | :---: |
| 34. | Direct proportion | Two quantities increase at the same rate <br> Graph is a straight line that goes through the origin | $y \propto x$ <br> $y=k x$ for a constant $k$ |
| 35. | Inverse/indirect proportion | One variable increases at a constant rate as the second variable decreases | $y \propto \frac{1}{x}$ <br> $y=\frac{k}{x}$ for a constant $k$ |
| 36. | Constant of proportionality | Represented by $k$ |  |
|  |  | Its value stays the same |  |

## Similarity and Congruence in 2D and 3D

| 1. | Congruent | Exactly the same shape and size |  |
| :---: | :--- | :--- | :--- | :--- |
| 2. | Similar | Two shapes where one is an enlargement of another |  |
|  |  | Corresponding angles are equal | Corresponding sides are in the same ratio |
| 3. | Scale factor | The proportion by which the dimensions of an object will increase or decrease by |  |
| 4. | Linear scale <br> factor (LSF) | The scale factor/ratio of sides of two <br> similar shapes | LSF $=\frac{\text { length from large shape }}{\text { length from small shape }}$ |
| 5. | Area scale <br> factor (ASF) | The scale factor ratio of areas/surface <br> areas of two similar shapes | ASF $=\frac{\text { Area of large shape }}{\text { lArea of small shape }}$ |
| 6. | Volume scale <br> factor (VSF) | The scale factor/ratio of volumes of two <br> similar shapes | $V S F=\frac{\text { volume of large shape }}{\text { volume of small shape }}$ |

Two triangles are congruent if...

| 7. | SSS | All 3 sides are equal |  |
| :---: | :---: | :---: | :---: |
| 8. | SAS | 2 sides and the included angle are equal |  |
| 9. | ASA | 2 angles and the corresponding side are equal |  |
| 10. | RHS | The right angle, hypotenuse and one other side are equal |  |

## Similar shapes

| 11. | Lengths | $\frac{\overline{E F}}{\overline{B C}}=\frac{12}{6} \div \frac{6}{6}=\frac{2}{1}=2 \frac{\overline{B C}}{\overline{E F}}=\frac{6}{12}=$ | The scale factor from small to big is 2. |
| :---: | :---: | :---: | :---: |
| 12. | Areas |  | $\begin{aligned} \mathrm{LSF} & =9 \div 6 \\ & =1.5 \\ \mathrm{ASF} & =1.5^{2} \end{aligned}$ <br> So area of bigger shapes is $6 \times 1.5^{2}$ |
| 13. | Volumes | Volume $=$ ? <br> Volume $-2500 \mathrm{~cm}^{3}$ | $\begin{aligned} \text { LSF } & =20 \div 8 \\ & =2.5 \\ U S F & =2.5^{2} \end{aligned}$ <br> So volume of smaller shape is $2500 \div \mathbf{2 . 5}{ }^{2}$ |

## Graph transformations

| 1. | $y=-f(x)$ | Reflection in the x axis | $y$ coordinates are multiplied by -1 |
| :---: | :---: | :--- | :--- |
| 2. | $y=f(-x)$ | Reflection in the y axis | x coordinates are divided by -1 |
| 3. | $y=-f(-x)$ | Reflection in the x axis and then in the y <br> axis | Equivalent to rotation of $180 \circ$ about the <br> origin |
| coordinates are multiplied by -1 AND x |  |  |  |
| coordinates are divided by -1 |  |  |  |$|$|  |
| :--- |
| 5. |
| 6. $y=f(x+a)$ |

## Exact Trig values

| 8. | Exact Values | $\theta$ | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\operatorname{Sin} \theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
|  |  | $\operatorname{Cos} \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
|  |  | $\operatorname{Tan} \Theta$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ |  |
|  |  | These can be found using the triangles: |  |  |  |  |  |

## Trigonometric graphs

9. 

Sine graph

|  |  | Crosses the x-axis at $-180^{\circ}, 0^{\circ}, 180^{\circ}, 360^{\circ} \ldots$ |  |
| :---: | :---: | :---: | :---: |
|  |  | Maximum of 1 and minimum of -1 | -270 |
| 10. | Cosine graph | Repeats every 360 |  |
|  |  | Crosses $x$-axis at -90 ${ }^{\circ} 90^{\circ}, 270^{\circ}, 450^{\circ}$... |  |
|  |  | Maximum of 1 and minimum of -1 |  |
| 11. | Tangent graph | Repeats every $180{ }^{\circ}$ |  |
|  |  | Crosses $x$-axis at -180 ${ }^{\circ} \mathbf{0}^{\circ}, 180{ }^{\circ}, 360^{\circ} \ldots$ |  |
|  |  | Has no maximum or minimum value |  |
|  |  | Has vertical asymptotes at $x=-90^{\circ}, x=90^{\circ}$, $x=270^{\circ}$... |  |

## Non - right angled trigonometry

| 12. | Cosine rule | Finding sides |  | Finding angles |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & a^{2}=b^{2}+c^{2}-2 b c \\ & b^{2}=a^{2}+c^{2}-2 a c \\ & c^{2}=a^{2}+b^{2}-2 a b \end{aligned}$ | s $A$ | $\begin{aligned} & \cos A=\frac{b^{2}+c^{2}-a^{2}}{2 b c} \\ & \cos B=\frac{a^{2}+c^{2}-b^{2}}{2 a c} \\ & \cos C=\frac{a^{2}+b^{2}-c}{2 a b} \end{aligned}$ |
| 13. | Sine rule | Finding sides $\frac{a}{\sin (A)}=\frac{b}{\sin (B)}=\frac{c}{\sin (C)}$ | Finding angles $\frac{\sin (A)}{a}=\frac{\sin (B)}{b}=\frac{\sin (C)}{c}$ | Ambiguous case Can sometimes produce two possible solutions for missing angles $\sin \theta=\sin (180-\theta)$ |


| 14. | Area of a triangle | $\begin{aligned} & \text { Area }=\frac{1}{2} a b \sin C \\ & \text { Area }=\frac{1}{2} b c \sin A \\ & \text { Area }=\frac{1}{2} a c \sin B \end{aligned}$ |  |
| :---: | :---: | :---: | :---: |

Collecting data

| 1. | Population | The whole set of items that are of interest e.g. all the people in a school |
| :---: | :--- | :--- | :--- | :--- | (

## Random sampling techniques

| 9. | Simple random sampling | Where every member of the sampling frame has an equal chance of being selected. |  |
| :---: | :---: | :---: | :---: |
|  |  | Advantages <br> - Free of bias <br> - Easy and cheap to implement for small populations and samples | Disadvantages <br> - Not suitable when population size or sample size is large <br> - A sampling frame is needed |


| 10. | Systematic sampling | Where required elements are chosen at regular intervals from an ordered list |  |
| :---: | :---: | :---: | :---: |
|  |  | Advantages <br> - Simple and quick to use <br> - Suitable for large samples and populations | Disadvantages <br> - A sampling frame is needed <br> - It can introduce bias if the sampling frame is not random |
| 11. | Stratified sampling | The population is divided into mutually exclusive strata (e.g. males and females) and a random sample is taken from each |  |
|  |  | Number sample in a stratum$=\frac{\text { number in stratum }}{\text { number in population }} \times \text { overall sample size }$ |  |
|  |  | Advantages <br> - Sample accurately reflects the population structure <br> - Guarantees proportional representation of groups within a population | Disadvantages <br> - Population must be clearly classified into distinct strata <br> - Selection within each stratum suffers from the same disadvantages as simple random sampling |
| Non- random sampling techniques |  |  |  |
|  |  | A researcher selects a sample that reflects the characteristics of the whole population |  |
| 12. | Quota sampling | Advantages <br> - Allows a small sample to be representative of the whole population <br> - No sampling frame required <br> - Quick, easy and inexpensive <br> - Allows for easy comparison between different groups in a population | Disadvantages <br> - Non random sampling can introduce bias <br> - Population must be divided into groups which can be costly or inaccurate <br> - Increasing scope of study increases number of groups, which adds time and expense <br> - Non-responses are not recorded as such |
| 13. | Opportunity sampling | Taking the sample from people who are available at the time the study is carried out and who fit the criteria you are looking for |  |
|  |  | Also known as 'convenience sampling' |  |
|  |  | Advantages <br> - Easy to carry out <br> - Inexpensive | Disadvantages <br> - Unlikely to provide a representative sample <br> - Highly dependent of the individual researcher |

## Types of data

| 14. | Quantitative data <br> (or variables) | Data (or variables) associated with numerical observations e.g. shoe size |
| :---: | :--- | :--- |
| 15. | Qualitative date <br> (or variables) | Data (or variables) associated with non-numerical observations e.g. hair colour |
| 16. | Continuous <br> variable (data) | A variable that can take any value in a given range e.g. time |
| 17. | Discrete variable <br> (data) | A variable that can take only specific values in a given range e.g. number of girls <br> in a family |

## Representing and interpreting data

| 18. | Class | Another name for the groups in a grouped frequency table |
| :---: | :--- | :--- |
| 19. | Class boundaries | The maximum and minimum values that belong in each class |
| 20. | Class width | The difference between the upper and lower class boundaries |
| 21. | Midpoint | The average of the class boundaries |
| 22. | Outlier | An extreme value that lies outside the overall pattern of the data |
| 23. | Anomalies | Any outliers that should be removed from the data because it is an error and it <br> would be misleading to keep it in |

## Types of graphs/charts



| 27. | Upper class boundary | The highest possible value in each class |  |
| :---: | :---: | :---: | :---: |
| 28. | Cumulative frequency graph | A graph with the data values on the $x$ axis and the cumulative frequency on the $y$ axis |  |
| 29. | Histogram | A chart where the area of each bar is proportional to the frequency of each class <br> Area of each bar $=k \times$ frequency $(k=$ 1 is the easiest value to use when drawing a histogram) |  |
| 31. | Frequency density | The height of each bar on a histogram | If $\boldsymbol{k}=\mathbf{1}$ then: $\text { frequency density }=\frac{\text { frequency }}{\text { class width }}$ |
| 31. | Frequency polygon | Can be formed by joining the middle of each bar in a histogram |  |

## Quadratics - definitions

| 2. | Roots | Solutions to a quadratic equation/function $a x^{2}+b x+c=0$ |  |
| :---: | :---: | :---: | :---: |
|  |  | The x values where the graph crosses the x axis |  |
|  |  | A quadratic can have 0,1 or 2 roots |  |
| 3. | Quadratic graph | Curved shaped called a parabola |  |
|  |  | A positive $x^{2}$ will give a ' $u$ ' shape |  |
|  |  | A negative $x^{2}$ will give a ' $n$ ' shape |  |
| 4. | Turning points | The point where a curve turns in the opposite direction |   |

## Using the discriminant

| 5. | Discriminant | The part of the quadratic formula under the <br> square root | $b^{2}-4 a c$ |
| :---: | :---: | :---: | :---: |
| 6. | $b^{2}-4 a c>0$ | Two distinct real roots |  |
| 7. | $b^{2}-4 a c=0$ | One repeated real root |  |
| 8. | $b^{2}-4 a c<0$ | No real roots |  |

## Skletching quadratic graphs

9. 

| General shape | A positive $x^{2}$ will give $a$ ' $U$ ' shape <br> A negative $x^{2}$ will give $a$ ' $n$ ' shape |  |
| :---: | :--- | :--- |
| Find the roots | By factorising or using the formula | Equation must be equal to zero |
| Find the $y$ <br> intercept | Substitute $x=0$ zero into the equation |  |
| Calculate the <br> coordinates of <br> the turning point | Complete the square to get in the form of <br> $\mathbf{f ( x ) = a ( x + p ) ^ { 2 } + \boldsymbol { q }}$ | Coordinates of turning point are <br> then $\quad(-p, q)$ |

## Solving quadratic inequalities

| 10. | Solve (by factorising or using quadratic formula) $a x^{2}+b x+c=0$ | e. 9 $\begin{gathered} x^{2}-2 x+8=0 \\ (x+4)(x-2)=0 \\ x=-4 \text { or } x=2 \end{gathered}$ |
| :---: | :---: | :---: |
| 11. | Sketch the graph clearings showing the roots and parabola shape |  |
| 12. | Check whether your quadratic was greater than or less than zero then highlight parts of the graphs that satisfy this | $x^{2}-2 x+8>0$  <br> Therefore $x<-4$ or $x>2$ is the solution |
|  |  |  |



| 15. | Cyclic <br> quadrilateral | A quadrilateral with all four vertices <br> on the circumference of a circle |  |
| :--- | :--- | :--- | :--- |

## Circle Theorems

| 16. | Angles at the centre | Angle at the centre is twice the angle at the circumference |  |
| :---: | :---: | :---: | :---: |
| 17. | Angles in the same segment | Angles at the circumference in the same segment are equal |  |
| 18. | Angles in a semicircle | Angle in a semi-circle is $90^{\circ}$ |  |
| 19. | Cyclic quadrilateral | Opposite angles of a cyclic quadrilateral add to $180^{\circ}$ |  |
| 20. | Tangent to a circle | Angle between a tangent and radius is $90^{\circ}$ <br> Two tangents from the same point to a circle are equal in length |  |


| 21. | Alternate <br> segment | Angles in the alternate segment are <br> equal |  |
| :---: | :--- | :--- | :--- |

## Circle geometry

| 22. | Equation of a circle | With centre $(0,0)$ and radius, $r$ $x^{2}+y^{2}=r^{2}$ | With centre ( $a, b$ ) and radius, $r$ $(x-a)^{2}+(y-b)^{2}=r^{2}$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 23. | Intersections between circles and lines | - No intersection <br> - Once (where the line touches the circle <br> - Twice (where the line crosses the circle) |  |
| 24. | Gradient of a radius to a circle | Gradient (m) of radius to a point $(x, y)$ with an equation $x^{2}+y^{2}=r^{2}$ is $\frac{y}{x}$ |  |
| 25. | Gradient of tangent to a circle | Gradient (m) of tangent to a point $(x, y)$ is the negative reciprocal of the gradient of the radius at the same point |  |

## Surds

| 1. | Surd | A number written exactly using square or cube roots | e.g. $\sqrt{ } 5$ is a surd but $\sqrt{25}$ is not because it has $a$ value of 5 |
| :---: | :---: | :---: | :---: |
| 2. | Rationalise | Eliminate a surd |  |
| 3. | Multiply | $\sqrt{a} \times \sqrt{b}=\sqrt{a b}$ and $\sqrt{a} \times \sqrt{a}=a$ | e.g. $\sqrt{2} \times \sqrt{3}=\sqrt{6}$ and $\sqrt{3} \times \sqrt{3}=3$ |
| 4. | Divide | $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ | e.g. $\frac{\sqrt{6}}{\sqrt{2}}=\sqrt{\frac{6}{2}}=\sqrt{3}$ |
| 5. | Add and subtract | $\sqrt{a}+\sqrt{b}$ cannot simplify | e.g. $\sqrt{3}+\sqrt{2}=\sqrt{3}+\sqrt{2}$ |
|  |  | But $\sqrt{a}+\sqrt{a}=2 \sqrt{a}$ | e.g. $5 \sqrt{2}-2 \sqrt{2}=3 \sqrt{2}$ |
| 6. | Simplify | $\sqrt{50}=\sqrt{25 \times 2}=\sqrt{25} \times \sqrt{2}=5 \sqrt{2}$ | e.g. $\sqrt{50}+\sqrt{18}=5 \sqrt{2}+3 \sqrt{2}=8 \sqrt{2}$ |
| 7. | Rationalise the denominator | Multiply numerator and denominator (use equivalent fractions) by whatever will result in the denominator simplifying to an integer. | $\text { e.g. } \frac{1}{\sqrt{7}}=\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}=\frac{7}{\sqrt{7}}$ |
|  |  |  | e.g. $\frac{1}{5+\sqrt{2}}=\frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}=\frac{5-\sqrt{2}}{3}$ |

## Algebraic Fractions

| 8. | Simplifying | Cancel common factors (factorising if <br> needed) | $\frac{(x-3)(x+2)}{(x+2)(x+5)}=\frac{x-3}{x+5}$ |
| :---: | :--- | :--- | :---: |
| 9. | Adding and <br> subtracting | Find a common denominator | $\frac{a}{b}+\frac{c}{d}=\frac{a d}{b d}+\frac{b c}{b d}=\frac{a d+b c}{b d}$ |
| 10. | Multiplying | Multiply as with normal fraction | $\frac{a}{b} \times \frac{c}{d}=\frac{a c}{b d}$ |
| 11. | Dividing | Divide as with normal fractions | $\frac{a}{b} \div \frac{c}{d}=\frac{a}{b} \times \frac{d}{c}=\frac{a d}{b c}$ |

## Changing the subject of a formula

Always use inverse operations to isolate the term you have been asked to make the subject

If the letter you want as the subject appears twice you will need to factorise
12.

Make $u$ the subject:
$v=u+a t$
$(-a t)$
$v-a t=u$
So
$u=v-a t$

$$
\begin{gathered}
\text { Make } u \text { the subject: } \\
v^{2}=u^{2}+2 a s \\
(-2 a s) \\
v^{2}-2 a s=u^{2} \\
(\sqrt{ }) \\
\sqrt{v^{2}-2 a s}=u \\
\text { So } \\
u=\sqrt{v^{2}-2 a s}
\end{gathered}
$$

Make $m$ the subject:
$I=m v-m u$
(Factorise)
$I=m(v-u)$
$(\div(\boldsymbol{v}-\boldsymbol{u}))$
$\frac{I}{v-u}=m$

So
$m=\frac{I}{v-u}$

## Algebraic proof

| 13. | Proof | A logical argument fro a mathematical statement |
| :---: | :--- | :--- |
|  |  |  |
| 14. | Counter <br> example | Use an example that does not fit the statement to prove the statement is incorrect |

## Notation to use in proof

| 15. | n | Any number |
| :--- | :--- | :--- |
| 16. | $\mathrm{n}+1$ | Consecutive number |
| 17. | 2 n | Even number |
| 18. | $2 \mathrm{n}+2$ | Consecutive even number to 2 n |
| 19. | $2 \mathrm{n}+1$ | Odd number |
| 20. | $2 \mathrm{n}+3$ | Consecutive odd number to $2 \mathrm{n}+1$ |
| 21. | an | A multiple of a e.g. 3n represents a multiple of 3 |

## Functions

| 22. | Function | A rule for working out values of $y$ (output) given values of $x$ (input) |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 23. | $f(x)$ | Function notation read as 'f of x ', where x is the input into the function |  |  |
| 24. | Composite functions | $f g(x)$ | Evaluate $g(x)$ first then substitute this into $f(x)$ |  |
| 25. |  | $g f(x)$ | Evaluate $f(x)$ first then substitute this into $g(x)$ |  |
| 26. | Inverse fuction | $f^{-1}(x)$ | Reverses the effect of the original function | $\begin{gathered} f(x)=3 x+2 \\ f^{-1}(x)=\frac{x-2}{3} \end{gathered}$ |

