

Year 9 Mathematics Higher HT 2

Algek	ora: the basics						
Defini	tions						
1.	Variable	A letter representing a varying or unknown quantity.					
2.	Coefficient	A number which multiplies a variab	le e.g. 4 is the coefficient in 4a				
	Term	One part of an expression/equation/	formula e.g. 4c				
3.		Can involve multiplying and dividinand variables	g coefficients W				
		Separated from other terms by add subtraction	3				
4.	Like terms	Terms that have the same variable but have different coefficients	e.g. $c + 4c$ are like terms $c^2 \text{ and } c^3 \text{ are not like terms}$				
		A fixed value.	Coefficient Variable				
5.	Constant	A number on its own or sometimes a letter such as a, b or c to represent a fixed number.	Operator Constants				
		One or a group of terms.					
6.	Expression	Can include variables, constants, operators and grouping symbols.	e.g. 3y -3 3y ² +y ³				
		No 'equals' sign	392+93				
7.	Equation	Contains an 'equals' sign, = Has at least one variable	e.g. 3y - 3 = 12				
8.	Formula	A special type of equation that show variables	vs the relationship between a set of				
9.	Formulae	Plural of 'formula'					
10.	Identity	An equation that is true no matter what values are chosen, \equiv e.g. $\exists y \equiv 2y - y$ for any value y .					
11.	Subject	The variable on its own on one side of the equals sign.					
12.	Substitute	Replace a variable with a number. $a = 3, b = 2 \text{ and } c = 5.$ Find: $1. 2a = 2 \times 3 = 6$ $2. 3a - 2b = 3 \times 3 - 2 \times 3$ $3. 7b^2 - 5 = 7 \times 2^2 - 5 = 3$					
13.	Simplify	Minimising the size of an expression	•				
-		•					

14.	Factorise Splitting an expression into a product of factors						
15.	Expand	Removing brackets by usin	ng multiplio	cation			
16.	Solve Find the value of an unknown						
Algebr	aic Notation	· I					
17.	Adding like terms	Add the coefficients		<i>b</i> -	+2b=3b		
18.	Subtracting like terms	Subtract the coefficients		5 <i>b</i>	-4b = b		
19.	Multiplying like terms	If the base is the same, ad powers	d the		$\times b = b^2$		
20.	Dividing terms	If the base is the same, sub powers	otract the	b^5	$\div b^2 = b^3$		
21.	Adding different terms	Cannot combine if the terr different.		b + 1	2c = b + 2c		
22.	Subtracting different terms	Cannot combine if the terr different.	ms are	3 <i>c</i> –	4 = 3c - 4		
23.	Multiplying different terms	Combine with no 'x' sign		$d \times e = de$			
24.	Multiplying different terms with coefficients	Combine with no 'x' sign, the coefficients	multiply	$2d \times 3e = d6e$			
25.	Dividing different terms	Write as fractions with no	'÷' sign	3 <i>d</i>	$\div e = \frac{3d}{e}$		
26.	Dividing different terms with coefficients	Write as fractions with no simplify the coefficients who possible.		14 <i>d</i>	$\div 7e = \frac{2d}{e}$		
xpan	nding (single brackets)						
27.			n the outsic	de.			
28.	\times $2x$ -3						
acto	rising (single brackets))					
	 Find the highest conterms This goes outside th 		2x +	+ 4y	2(x + 2y)		
29.	Divide each term be new terms inside th	y the factor to get the	5x ² y –	- 10xy	5xy(x - 2)		
_	ssions						

Can be represented by a straight

An expression where the highest

No indices above 1

index is 2

30.

31.

Linear

Quadratic

e.g. 2x + 2

e.g. $2x^2 + 2x + 2$

Expanding double brackets

32. Everything in the first bracket must be multiplied by everything in the second

Grid method
(x+4)(x+7)
x x +4 · · x x² 4x
+7 Be 28
$= x^2 + 4x + 7x + 28$
$= x^2 + 11x + 28$

FIRST: (x+3)(x-4) gives $x \times x = x^2$ OUTER: (x+3)(x-4) gives $x \times (-4) = -4x$

FOIL method

INNER:
$$(x+3)(x-4)$$
 gives $3 \times x = 3x$
LAST: $(x+3)(x-4)$ gives $3 \times (-4) = -12$

Factorising a quadratic expression

		Multiply to 5	
		Factorise $x^2 + 5x + 6 \leftarrow Add$ to 6	
24	Factorising a	2 and 3 add to 5	
34.	quadratic in the form of $ax^2 + bx + c$	2 and 3 multiply to 6	
		(x+2)(x+3)	
		Check: $(x+2)(x+3) = x^2 + 5x + 6$	
		A special type of quadratic which only has two terms.	
	Difference of two	One term is subtracted from the other	
35.	squares	$x^2 - 25 = x^2 - 5^2 = (x + 5)(x - 5)$	
		$y^2 - 49 = y^2 - 7^2 = (y + 7)(y - 7)$	
		$a^2 - 16 = a^2 - 4^2 = (a + 4)(a - 4)$	

Equations

33.

- 36. To solve equations we need to use inverse operations
- 37. What ever you do to one side of the equals sign you must do the same to the other

	T						
38.	One step		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$				
39.	Two step	Requires the use of two inverse operations	2x - 7 = 19 $2x = 26$ $x = 13$				
40.	With brackets	Expand the brackets first $5(2x + 1) = 35$ $10x + 5 = 35$ $10x = 30$ $x = 3$	OR if possible divide by the number outside of the bracket first $4(2x + 4) = 20$ $2x + 4 = 5$ $2x = 1$ $x = \frac{1}{2}$				
41.	Unknowns on both sides	Start by eliminating the unknown from one of the signs.	5x + 2 = 3x - 8 $2x + 2 = -8$ $2x = -10$ $x = -5$				
42.	With fractions	Eliminate any terms that are being added or subtracted separate from the fraction first. $\frac{f}{5}+2=8$ $\frac{f}{5}=6$ $f=30$	If everything is part of the fraction then multiply by the denominator first. $\frac{f+2}{5}=8$ $f+2=40$ $f=38$				
Chang	Changing the subject of a formula (rearranging)						

anging the subject of a formula (rearranging)

Always use inverse operations to isolate the term you have been asked to make the subject

If the letter you want as the subject appears twice you will need to factorise

43.	Make u the subject: v = u + at (-at) v - at = u So	Make u the subject: $v^{2} = u^{2} + 2as$ $(-2as)$ $v^{2} - 2as = u^{2}$ $(\sqrt{})$ $\sqrt{v^{2} - 2as} = u$	Make m the subject: $I = mv - mu$ $(Factorise)$ $I = m(v - u)$ $(\div (v - u))$ $\frac{I}{v - u} = m$
	u = v - at	So $u = \sqrt{v^2 - 2as}$	$m = \frac{I}{v - u}$

Iterati	ion					
44.	Iteration	The act of repeating a process to generate a sequence of outcomes or with the aim of of appraoching a desired result e.g. finding a solution to an equation				
45.	Iterative sequence	The relationship between consecutive terms				
46.	Roots	Solutions to an equation				
47.	Change of sign	Two values with a root between them				
Seque	ences					
48.	Sequence	An order pattern of numbers or diagrams				
49.	Term	One of the numbers or diagrams in a sequence				
50.	Term to term rule	The rule for moving from one term to the next in a sequence				
51.	Formula	A rule written to describe a realtionship between twp quantities				
52.	Arithmetic sequence	A sequence where the term to term rule is to addd or subtract the same amount each time				
53.	Quadratic	A sequence where the term to term rule is changing by the same amount each time				
55.	sequence	The second difference is a constant amount.				
54.	Geometric sequence	A sequence where the term to term rule is to multiply by the same amount each time				
55.	Common	The value a geometric sequence is multiplied by from one term to the next				
<i>55.</i>	ratio	Denoted by the letter <i>r</i>				
56.	Series	The sum of the terms in a sequence				
57.	Position to term rule	The rule for finding any value of a sequence				
		The rule to find any term in a sequence of numbers				
58.	nth term rule for an arithmetic sequence	 Find the common difference between the terms This becomes you coefficient of n (this is the times table the sequenc is linked to) The number you need to add or subtract to get to the second term becomes the second term in the nth term rule Now compare the sequence to the 4 times table 6, 10, 14, 18, 22 Each term is 2 bigger than the 4 times table increases by 4, so the nth term starts with 4n A so the nth term is 				
59.	Nth term for a quadratic sequence	 Find the first difference Find the second difference Halve the second difference and multiply by n² to gain a new sequence of an² Generate the first few term sof this seuence then subtract from the original sequence 				

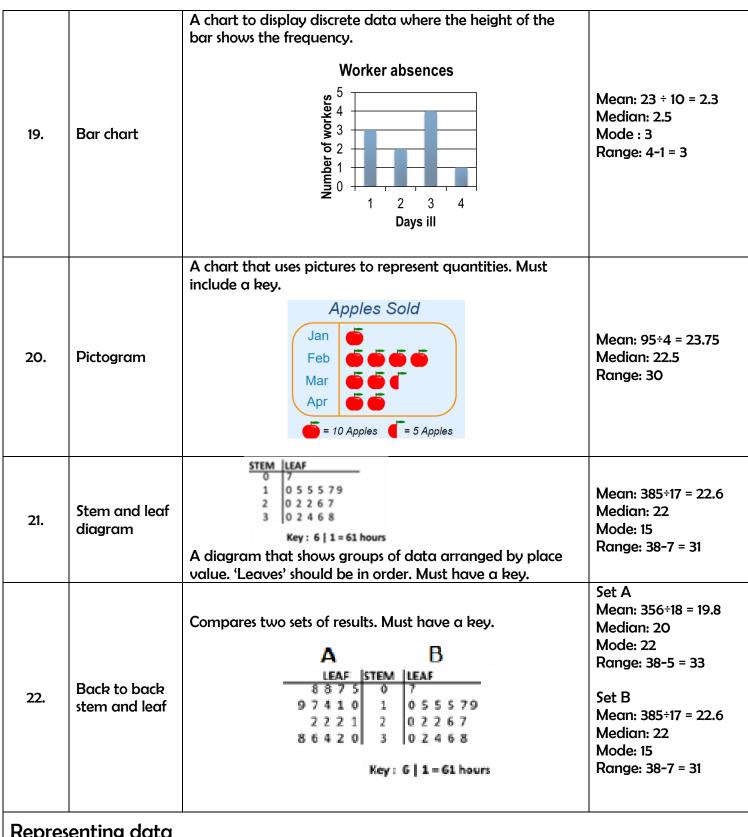
		• Find the nth term of the remianing sequence $bn+c$ • The entire nth term is then an^2+bn+c					
60.	nth term for a geometric sequence	 Divide the second sequence by the first to find the common ratio, r The nth term is arⁿ⁻¹ where a is the first term and n is the term position in the sequence 					
61.	Finite	Has a f	inal point				
62.	Infinite	Carries	on forever				
63.	Ascending	Increase	es				
64.	Descending	Decrea	ses				
65.	Linear function	An aru	thmetic sequence that can be represent	ed by a straight line graph			
Special	Sequences						
66.	Square numbers		1, 4, 9, 16, 25, 36, 49, 64, 81, 100	1 4 9 16			
67.	Cube numbers		1, 8, 27, 64, 125	1 8 27 64 125			
68.	Triangular numbers		1, 3, 6, 10, 15, 21, 28				
	Ethan and		A sequence where each term is the sur	n of the two previous terms			
69. Fibonacci sequence		ence	e.g. 1, 1, 2, 3, 5, 8, 13, 21				



Year 9 Mathematics Higher HT 3

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Defin	itions					
1.	Qualitative Data	Non-numerical data i.e. Colour of car				
2.	Quantitative Data	Numerical data	i.e. House number			
3.	Discrete Data	Numerical data that <u>CANNOT</u> be shown in decimals	i.e. Number of children in a class			
4.	Continuous Data	Numerical data that <u>CAN</u> be shown in decimals	i.e. The heights of children in a class			
5.	Grouped Data	Numerical data given in intervals	i.e. Year group ranges: Year 7-9 Year 10-11 Year 12-13			
Averd	ages					
6.	Measure of location	A single value that describes a position in a	data set			
7.	Measure of central tendency	A single value that describes the centre of the data				
		A measure of how spread out the data is				
8.	Measure of spread	Also known as 'measures or dispersion' or 'measures of variation'				
		Two simple measures of spread are range and interquartile range (IQR)				
9.	Mode (modal class)	The value that occurs most often				
10.	Range	The difference between the largest and sma	llest values in the data set			
11.	Median	The middle value when the data values are	put in ascending order			
		Found by adding all number sin the data se in the set	t and dividing by the number of values			
12.	Mean	Can be calculate using the formula $\bar{x} = \frac{\Sigma x}{n}$ Mean from a frequency table	There: \bar{x} is the mean Σx is the sum of the data values n is the number of data values			
ız.	Wedi	$\bar{x} = \frac{\Sigma}{2}$	$\frac{\Sigma f x}{\Sigma f}$			
		Where $\Sigma f x$ is the sum of the products of dais the sum of the frequencies	-			

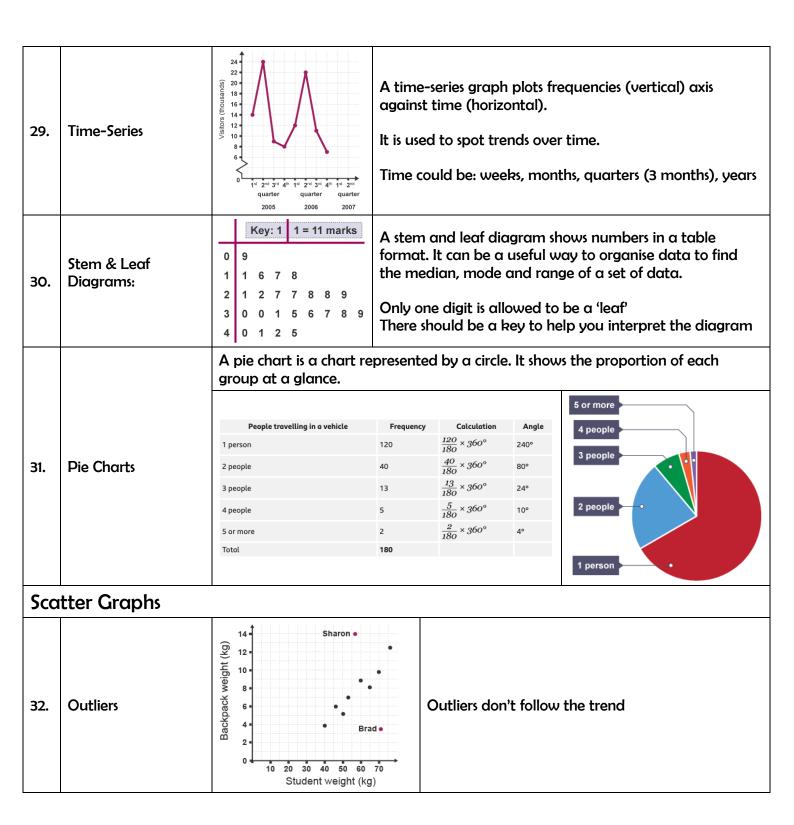
	Average	Advantages				Disadvantages	
	Mean	Every value makes a difference			A	Affected by extreme values	
13.	Median	Not affected by	extreme	values	М		hange even if a data value
	Mode	Easy to find; not affected by extreme values; can be used for non-numerical data				There may not be a mode	
Avera	ges from freque	ncy tables					
14.	Modal class	The class with the	e highest	frequen	су		
15.	Median	If the total freque	ency is n	, then the	e median	lies in th	e class with the $\frac{n+1}{2}$ th value i
16.	Mean from a frequency table Times Add Divide		of make-up of Items Fr 1 7 2 2 3 1 4 4 5 2	1 x 7 2 2x 2 3x 1 4x 4 5x 2	=7 =4 =3 =16		Mean = $\frac{40}{16}$ = 2.5
17.	Estimated mean from a grouped frequency table Times Add Divide	Class Interval $140 \le h < 150$ $150 \le h < 160$ $160 \le h < 170$ $170 \le h < 180$	Mid-point 145 155 165 175 Totals	Frequency 6 16 21 8 51	155 × 16 165 × 21	= 870 = 2480	Mean = 8215 ÷ 51 =161.07843 = 161.08 (2dp)
18.	Estimate of range from grouped frequency table	The maxiumum	possible v	value mi	nus the sm	nallest p	ossible value.



Representing data

			Boys	Girls	TOTAL	Tours outside the same of outside of
23. Two-Way Tables	Two-Way	Pet	9	4	13	Two-way tables are a way of
	Tables	No Pet	2	5	7	sorting data with two
		TOTAL	11	9	20	categories.

24.	Pictograms	Movie genre Frequency Horror Action Romance Comedy Other = 4 people = 3 people = 2 people = 1 person	Used to show frequencies Pictures and images used to represent frequency A key at the bottom helps you interpret the diagram
25.	Bar Charts	15 10 10 10 10 10 10 10 10 10 10 10 10 10	Frequency on the vertical axis, and categories along the horizontal axis. Used to compare frequencies
26.	Composite Bar Chart	Number of pets Boys Boys	Frequency on the vertical axis, and categories along the horizontal axis. Two shades used to show difference in proportion between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
27.	Comparative Bar Chart	So Rainfall 40 30 Cm 20 Jan Feb Mar Apr May Month Dual Bar Chart	Frequency on the vertical axis, and categories along the horizontal axis. Bars are next to each other and used to show difference in frequency between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
28.	Line Graph	23 23 24 25 25 25 25 25 25 25 25 25 25 25 25 25	A line graph is used to show a change or relationship between two variables. Once the points are plotted, they are joined with straight lines.



33.	Line of Best Fit	50	possible through the p	e that goes as centrally as points plotted. The same steepness of the
34.	Interpolate	50	our range For example: To estimate sold with 3mm rai Find where 3 mm	to estimate data WITHIN nate how many umbrellas in. of rainfall is on the graph. ing across from 3 mm and
35.	Extrapolate	80 75 70 65 60 90 55 90 90 45 10 40 30 25 20 15 10 12 33 45 67 89 10 10 10 10 10 10 10 10 10 10	BEYOND our range (interpolation) For example: To estimate sold with 10mm range (interpolation) Continue the line of Find where 10mm	nate how many umbrellas ain.
36.	Positive Correlation	102 98 94 98 98 98 98 98 98 98 98 86 X X X X X X X X X X X X X	BOTH variables increase with each other	i.e. Ice creams sold vs Temperature
37.	Negative Correlation	plos signo o po seguino de la companya de la compan	ONE variable increases as the other decreases	i.e. Coats sold vs temperature

38.	No Correlation	x x x x x x X X X X X X X X X X X X X X	NO relationship between variables	i.e. IQ and House Number	
39.	Causation	 If one variable causes a change in the other. i.e. an increase temperature <u>WILL</u> cause an increase ice cream sales i.e. the number of bee stings <u>WILL NOT</u> cause an increase in ice cream sales (although both will increase in hot weather) 			

Fra	ctions					
1.	Fraction	Part of a whole				
2.	Numerator	The number on the top of the fraction	numerator			
3.	Denominator	The number on the bottom of the fro	action	denominator		
4.	Equivalent fractions	Fractions that have the same value look different.	but	$\frac{1}{2} \frac{2}{4} \frac{3}{6} \frac{4}{8}$		
5.	Improper fraction	A fraction where the numerator is la than the denominator.	arger	e.g. $\frac{4}{3}$		
6.	Mixed number	A number made from integer and fr	raction	e.g. $2\frac{2}{3}$		
7.	Unit fraction	A fraction that has a numerator of 1				
	Desirovesal	The reciprocal of a number is 1 divided by the number.	e.g. the	reciprocal of 3 is $\frac{1}{3}$		
8.	Reciprocal	Dividing by a number is the same as multiplying by its reciprocal	e.g. × by	by $\frac{1}{3}$ is the same as \div by 3		
Fra	ctions - processes		1			
9.	Simplifying fractions	Divide the numerator and denomine by the HCF.	ator	$\frac{24}{30} = \frac{4}{5}$		
10.	Finding equivalent fractions	Multiply the numerator and denominator by the same number		$\frac{4}{8} \times 2 = 8$ $\times 2 = 16$		
11.	Comparing fractions	Write them with a common denomin	nator			
12.	Fraction of an amount	Amount divided by the denominato then multiplied by the numerator	or	e.g. $\frac{5}{7}$ of 42 42 ÷ 7 x 5 = 30		
13.	Multiply fractions	Multiply the numerators and multiple the denominators	ly	$\frac{6}{7} \times \frac{4}{5} = \frac{6 \times 4}{7 \times 5} = \frac{24}{35}$		
14.	Divide fractions	 Flip the second fraction (find the reciprocal). Change the divide to multiply. Multiply the fractions. 		$\frac{4}{7} \div \frac{5}{6} = \frac{4}{7} \times \frac{6}{5} = \frac{4 \times 6}{7 \times 5} = \frac{24}{35}$		
15.	Add or subtract fractions	 Write as fractions with a common denominator. Add or subtract the numerat 	$\frac{2}{8}$	$\frac{1}{6} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$		
16.	Convert improper fractions to mixed numbers	 Divide the numerator by the denominator The answer gives the whole number part. 	}	$\frac{43}{6} = 7\frac{1}{6}$		

		-				
		The remainder becomes the numerator of the fraction part with the same denominator.				
17.	Convert mixed numbers to improper fractions	 Multiply the denominator by the whole number part. Add the numerator to this. Put the answer to this back over the denominator 	$7\frac{1}{6} = \frac{6 \times 7 + 1}{6} = \frac{43}{6}$			
18.	Adding and subtracting mixed numbers	 Convert mixed numbers to improp Transform both fractions so they he Add or subtract the numerators Convert back to mixed number if 	ave the same denominator			
19.	Multiplying mixed numbers	 Convert mixed numbers to improp Multiply numerators and multiply Convert back to mixed number if 	the denominators			
20.	Dividing mixed numbers	 Convert back to mixed number if applicable Convert mixed numbers to improper fractions Flip the second fraction (find the reciprocal) Change the divide sign to a multiply Multiply the fractions Convert back to mixed number if applicable 				
Per	centages					
21.	Percentage	Means 'out of 100'				
22	NA. Ikin lian	A decimal you multiply by to represent a percentage				
22.	Multiplier	To use a multiplier to find a percentage, divide your percentage by 100, then multiply the amount by this value.				
		Calculate the percentage and add onto the original				
23.	Percentage increase	Or use a multiplier	$amount \times \frac{100 + \% increase}{100}$			
		Calculate the percentage and subtract fro	om the original			
24.	Percentage decrease	Or use a multiplier	$amount \times \frac{100 - \% increase}{100}$			
25.	Percentage change	$\frac{Change}{Original} \times 100$				
26.	Express one number as a percentage of another	$\frac{Number\ 1}{Number\ 2} \times 100$				
27.	Reverse percentage	Use when asked to find the priginal amount after a percentage increase or decrease.				
-						

		Original Value x Multiplier = New Value				
		Original Value = <u>New Value</u> Multiplier				
28.	Interest	A fee paid for borrowing money or money earnt through investing.				
			I = Prt			
29.	Simple interest	Interest that is calculated as a percentage of the original	I – Interest P – Original amount r – interest rate t - time			
		When interest is calculate on the original amount and any previous interest	$P\left(1+\frac{R}{100}\right)^n$			
30.	Compound interest	OR	P – Original amount R – Interest rate			
		Original × Multiplier ^{time}	n – the number of interest periods (e.g. yrs)			
31.	Тах	A financial charge placed on sales or savings by the government e.g. VAT				
32.	Loss	Income minus all expenses, resulting in a negative value				
33.	Profit	Income minus all expenses, resulting in a positive value				
34.	Depreciation	A reduction in the value of a product over time				
35.	Annual	Means yearly				
36.	Per annum	Means per year				
37.	Salary	A fixed regular payment, often paid	monthly			
FDI	P Conversions	T				
38.	Percentage to decimal	Divide by 100				
39.	Decimal to percentage	Multiply by 100				
40.	Fraction to percentage	Find an equivalent fraction with 100	as the denominator			
41.	Percentage to fraction	Write as a fraction over 100 then simplify				
42.	Fraction to decimal	Carry out division or convert to a percentage first				

43.	Decimal to fra	ction		Use place value to find the denominator and simplify or convert to a percentage first								
Bas	ics to memo	rise										
		1		1	1	1	1	-	1	1	2	3
	Fraction	$\frac{10}{10}$	$\overline{00}$ $\overline{10}$		8	<u>-</u>	$\frac{1}{4}$	_	3	$\frac{\overline{2}}{2}$	3	$\frac{\overline{4}}{4}$
44.	Decimal	0.0		0.1	0.125	0.2	0.2		0. 3	0.5	0. 6	0.75
	Percentage	19	6	10%	12.5%	20%	25	%	33. 3%	50%	66. 7%	75 %
Ter	minating an	d re	ecur	ring de	cimals							
45.	Terminating decimal	Dec	imo	als that c	an be wr	itten exa	ctly	e.g.	. 0.38			
46.	Recurring			nals where one digit or groups				e.g.	. 0. 7 = o .	7777		
70.	decimal	of c	digit	s are rep	eated			0. Ė	353 = 0.8	53853		
			3. 4.	 Let n = the number of recurring digits. Multiply the recurring decimal by 10ⁿ. Subtract (1) from (3) to eliminate the recurring part. Solve for x, expressing your answer as a fraction in its simplest form. 								
	Converting a			0.7 (one recurring digit)			t)		1.256 (two recurring digits)			
47.	recurring decimal				= 0.7777			x=1.25656				
	to a fraction				= 7.777					=125.6565		
				10x - x = 9x						=125.656: =124.4	51.2565	65
				<i>J. X</i> ·	- <i>7</i>				99.1		1244 622	
				<i>x</i> :	$=\frac{1}{9}$				х	$=\frac{12.11}{99}$	$\frac{1211}{990} = \frac{922}{495}$	
	Converting a							e.g.	$\frac{4}{7}$	means 4 ÷ 7		
48.	Converting a fraction to recurring decimals			Carry out the neccesary division using a			ing a	,				
			calc	ualtor or I	ous stop div	vision		0.57142857				_
									/	4. 0 0 0	³ 0 ² 0 ⁶ 0 ⁴ 0 ⁵ 0	0
Rat	io and Prop	orti	on									
49.	Ratio		A	relations	hip betwee	en two or r	nore c	quant	tities			
F.0	. Unit ratio		U	Ised to co	mpare ratio	os, one of t	he pa	ırts is	1			
50	. Onit fatio		Т	The only time it is permissible to have a decimal in a ratio								

51.	Equivalent	Ratios that have the same simplified form are said to be equivalent				
52.	Scale	A ratio that represents the relationship between a length on a drawing or a map and the actual length				
53.	Proportion	Compares a part with a whole				
54.	Direct proportion	Two quantities increase at the same rate Graph is a straight line that goes through the origin	$y \propto x$ $y = kx$ for a constant k			
55.	Inverse/indirect proportion	One variable increases at a constant rate as the second variable decreases	$y \propto \frac{1}{x}$ $y = \frac{k}{x} \text{ for a constant } k$ $y = \frac{k}{x}$			
56.	Proportional	A change in one is always accompanied by a change in the other				
57.	Constant of proportionality	Represented by <i>k</i> Its value stays the same				
58.	Share	Splitting into parts as defined by a ratio				
59.	Unitary method	Finding the value of 1 item then using this to find the value of any number of that item Use to work out which products give the best value for money				
Work	ing with ratio	OS				
60.	Simplifying ratio	Divide all parts by the highest common factor All parts in the simplified version must be integers	e.g. 12:4 simplifies to 3:1 (divided by HCF of 4)			
61.	Divide in a given ratio	Divide an amount so the ratio of the final values simplifies to the given ratio	share £20 in the ratio 3:2 £20 £4 £4 £4 £4 £4			