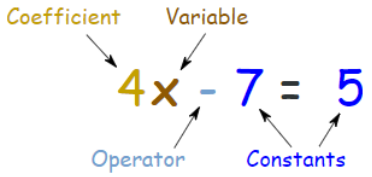
Year 9 Mathematics Foundation HT 2

Algebra: the basics

Definitions

1.	Variable	A letter representing a varying or unknown quantity.	
2.	Coefficient	A number which multiplies a variable e.g. 4 is the coefficient in 4a	
3.	Term	One part of an expression/equation/formula Can involve multiplying and dividing coefficients and variables Separated from other terms by addition and subtraction	e.g. $4c$ $\frac{w}{5}$
4.	Like terms	Terms that have the same variable but have different coefficients	e.g. c + 4c are like terms c^2 and c^3 are not like terms
5.	Constant	A fixed value. A number on its own or sometimes a letter such as a, b or c to represent a fixed number.	
6.	Expression	One or a group of terms. Can include variables, constants, operators and grouping symbols. No 'equals' sign	e.g. 3y -3 $3y^2 + y^3$
7.	Equation	Contains an 'equals' sign, = Has at least one variable	e.g. 3y – 3 = 12
8.	Formula	A special type of equation that shows the relationship between a set of variables	
9.	Formulae	Plural of 'formula'	
10.	Identity	An equation that is true no matter what values are chosen, ≡	e.g. 3y ≡ 2y – y for any value of y.
11.	Subject	The variable on its own on one side of the equals sign.	
12.	Substitute	Replace a variable with a number.	$a = 3, b = 2$ *and* $c = 5$. **Find:** 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$
13.	Simplify	Minimising the size of an expression	
14.	Factorise	Splitting an expression into a product of factors	
15.	Expand	Removing brackets by using multiplication	

16.	Solve	Find the value of an unknown	
Algebraic Notation			
17.	Adding like terms	Add the coefficients	$b + 2b = 3b$
18.	Subtracting like terms	Subtract the coefficients	$5b - 4b = b$
19.	Multiplying like terms	If the base is the same, add the powers	$b \times b = b^2$
20.	Dividing terms	If the base is the same, subtract the powers	$b^5 \div b^2 = b^3$
21.	Adding different terms	Cannot combine if the terms are different.	$b + 2c = b + 2c$
22.	Subtracting different terms	Cannot combine if the terms are different.	$3c - 4 = 3c - 4$
23.	Multiplying different terms	Combine with no '×' sign	$d \times e = de$
24.	Multiplying different terms with coefficients	Combine with no '×' sign, multiply the coefficients	$2d \times 3e = d6e$
25.	Dividing different terms	Write as fractions with no '÷' sign	$3d \div e = \frac{3d}{e}$
26.	Dividing different terms with coefficients	Write as fractions with no '÷' sign, simplify the coefficients where possible.	$14d \div 7e = \frac{2d}{e}$
Expanding (single brackets)			
27.	Multiply all the terms inside the bracket, by the term on the outside.		
28.	$3(a + 4) = 3a + 12$	× $2x$ / $2x$: $4x^2$, $-6x$	$4x^2 - 6x$
Factorising (single brackets)			
29.	• Find the highest common factor of the terms • This goes outside the bracket • Divide each term by the factor to get the new terms inside the bracket • Always check by expanding your bracket	$2x + 4y$ → $2(x + 2y)$ $5x^2y - 10xy$ → $5xy(x - 2)$	
Expressions			
30.	Linear	Can be represented by a straight line No indices above 1	**e.g.** $2x + 2$
31.	Quadratic	An expression where the highest index is 2	**e.g.** $2x^2 + 2x + 2$

Year 9 Mathematics Foundation HT 3

3a. TABLES

1	TYPES OF DATA		
1a	Qualitative Data	Non-numerical data	i.e. Colour of car
1b	Quantitative Data	Numerical data	i.e. House number
1c	Discrete Data	Numerical data that CANNOT be shown in decimals	i.e. Number of children in a class
1d	Continuous Data	Numerical data that CAN be shown in decimals	i.e. The heights of children in a class
1e	Grouped Data	Numerical data given in intervals	i.e. Year group ranges: Year 7-9 Year 10-11 Year 12-13

2 Data Collection

Three Columns:
- Data values
- Tally
- Frequency (how many)

Colour of Car	Tally	Frequency
Red	IIII	4
White	~~IIII~~	5
Grey	III	3
Black	~~IIII~~ III	8
Blue	~~IIII~~ I	6

3 Mode

Most popular (i.e. most common colour of car is Black)

4a	24-hour clock	15:30 — Uses hours 00 (Midnight) to 23 (11 in the evening)
4b	12-hour clock	03:30 pm — Uses hours 01 to 12 with morning (AM) and afternoon/evening (PM)

5 Two-Way Tables

	Boys	Girls	TOTAL
Pet	9	4	13
No Pet	2	5	7
TOTAL	11	9	20

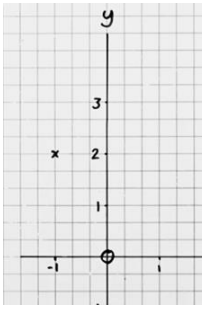
Two-way tables are a way of sorting data with two categories.

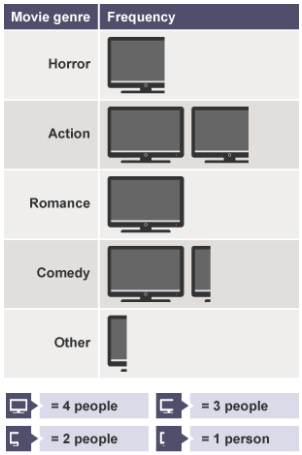
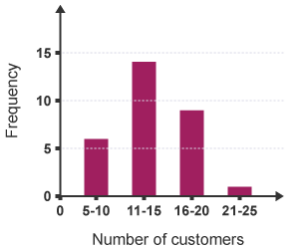
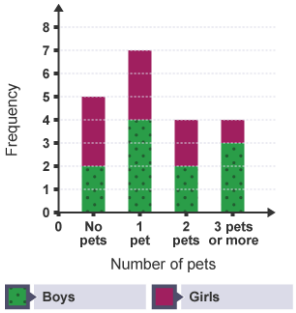
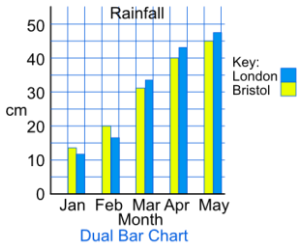
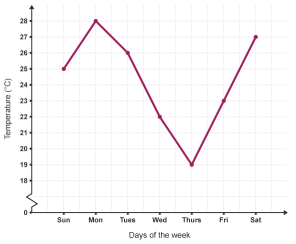
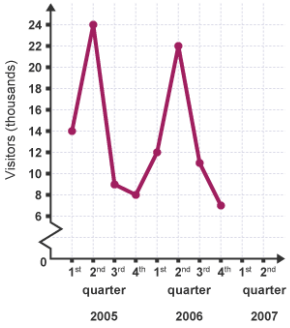
3b. Charts and Graphs

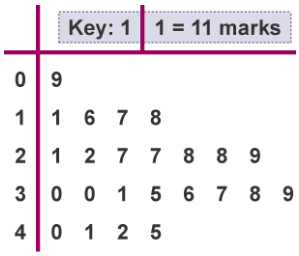
6 Plotting Points

Co-ordinates show an exact position

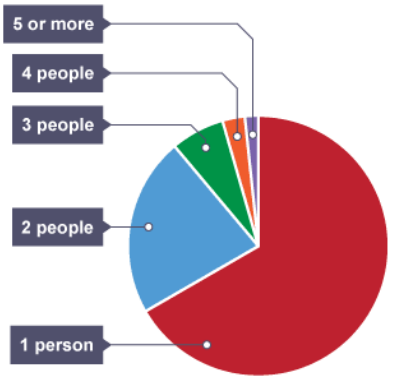
(x, y)



7	Pictograms		Used to show frequencies Pictures and images used to represent frequency A key at the bottom helps you interpret the diagram
8a	Bar Charts		Frequency on the vertical axis, and categories along the horizontal axis. Used to compare frequencies
8b	Composite Bar Chart		Frequency on the vertical axis, and categories along the horizontal axis. Two shades used to show difference in proportion between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
8c	Comparative Bar Chart		Frequency on the vertical axis, and categories along the horizontal axis. Bars are next to each other and used to show difference in frequency between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
9	Line Graph		A line graph is used to show a change or relationship between two variables. Once the points are plotted, they are joined with straight lines.
10	Time-Series		A time-series graph plots frequencies (vertical) axis against time (horizontal). It is used to spot trends over time. Time could be: weeks, months, quarters (3 months), years

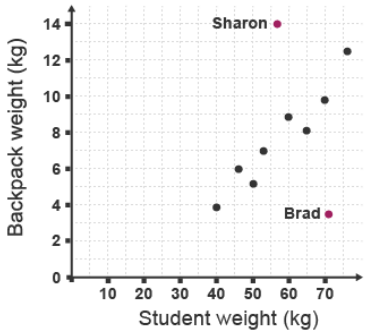
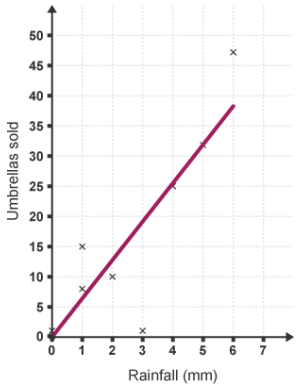
11	Stem & Leaf Diagrams:		A stem and leaf diagram shows numbers in a table format. It can be a useful way to organise data to find the median, mode and range of a set of data. Only one digit is allowed to be a 'leaf' There should be a key to help you interpret the diagram

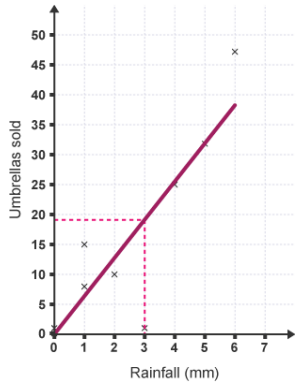
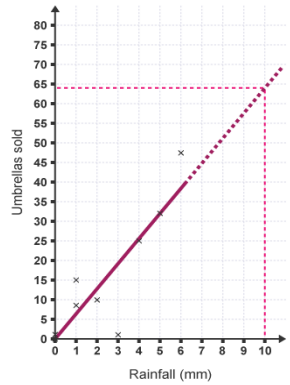
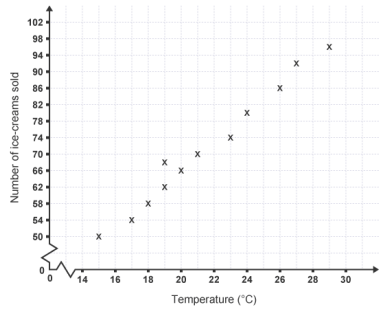
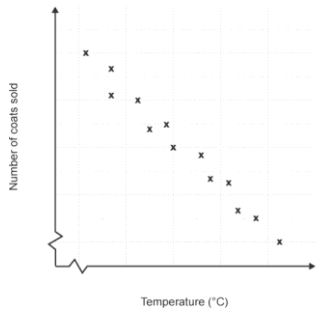
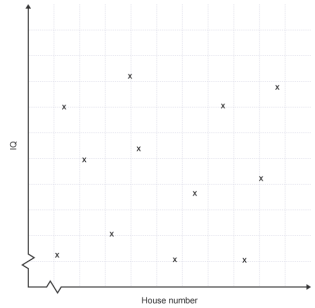
3c. Pie Charts

12	Pie Charts	A pie chart is a chart represented by a circle. It shows the proportion of each group at a glance. To find the angle: $\frac{frequency}{total} \times 360°$	

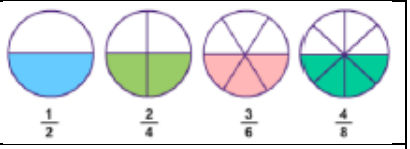
People travelling in a vehicle	Frequency	Calculation	Angle
1 person	120	$\frac{120}{180} \times 360°$	240°
2 people	40	$\frac{40}{180} \times 360°$	80°
3 people	13	$\frac{13}{180} \times 360°$	24°
4 people	5	$\frac{5}{180} \times 360°$	10°
5 or more	2	$\frac{2}{180} \times 360°$	4°
Total	180		

3d. SCATTER GRAPHS

13	Outliers		Outliers don't follow the trend
14	Line of Best Fit		A sensible straight line that goes as centrally as possible through the points plotted. It should also follow the same steepness of the crosses.

15	Interpolate		Using a line of best fit to estimate data <u>WITHIN</u> our range <u>For example</u>: To estimate how many umbrellas are sold with 3mm rain. • Find where 3 mm of rainfall is on the graph. • Draw a line by going across from 3 mm and then down.	
16	Extrapolate		Continuing a line of best fit to estimate data <u>BEYOND</u> our range (not as reliable as interpolation) <u>For example</u>: To estimate how many umbrellas are sold with 10mm rain. • Continue the line of best fit. • Find where 10mm of rainfall is on the graph. • Draw a line by going across from 10mm and then down.	
17a	Positive Correlation		BOTH variables increase with each other	i.e. Ice creams sold vs Temperature
17b	Negative Correlation		ONE variable increases as the other decreases	i.e. Coats sold vs temperature
17c	No Correlation		NO relationship between variables	i.e. IQ and House Number
18	Causation	If one variable causes a change in the other. • i.e. an increase temperature <u>WILL</u> cause an increase ice cream sales • i.e. the number of bee stings <u>WILL NOT</u> cause an increase in ice cream sales (although both will increase in hot weather)		

Fractions

1.	Fraction	Part of a whole	
2.	Numerator	The number on the top of the fraction	$\frac{numerator}{denominator}$
3.	Denominator	The number on the bottom of the fraction	
4.	Equivalent fractions	Fractions that have the same value but look different.	
5.	Improper fraction	A fraction where the numerator is larger than the denominator.	e.g. $\frac{4}{3}$
6.	Mixed number	A number made from integer and fraction parts.	e.g. $2\frac{2}{3}$
7.	Unit fraction	A fraction that has a numerator of 1	
8.	Reciprocal	The reciprocal of a number is 1 divided by the number.	e.g. the reciprocal of 3 is $\frac{1}{3}$
		Dividing by a number is the same as multiplying by its reciprocal	e.g. $\times$ by $\frac{1}{3}$ is the same as $\div$ by 3

Fractions - processes

9.	Simplifying fractions	Divide the numerator and denominator by the HCF.	$\frac{24}{30} = \frac{4}{5}$
10.	Finding equivalent fractions	Multiply the numerator and denominator by the same number	$\frac{4}{8} \times 2 = 8$ $\times 2 = 16$
11.	Comparing fractions	Write them with a common denominator	
12.	Fraction of an amount	Amount divided by the denominator then multiplied by the numerator	e.g. $\frac{5}{7}$ of 42 $42 \div 7 \times 5 = 30$
13.	Multiply fractions	Multiply the numerators and multiply the denominators	$\frac{6}{7} \times \frac{4}{5} = \frac{6 \times 4}{7 \times 5} = \frac{24}{35}$
14.	Divide fractions	• Flip the second fraction (find the reciprocal). • Change the divide to multiply. • Multiply the fractions.	$\frac{4}{7} \div \frac{5}{6} = \frac{4}{7} \times \frac{6}{5} = \frac{4 \times 6}{7 \times 5} = \frac{24}{35}$
15.	Add or subtract fractions	• Write as fractions with a common denominator. • Add or subtract the numerators	$\frac{2}{8} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$
16.	Convert improper fractions to mixed numbers	• Divide the numerator by the denominator • The answer gives the whole number part.	$\frac{43}{6} = 7\frac{1}{6}$

		• The remainder becomes the numerator of the fraction part with the same denominator.	
17.	Convert mixed numbers to improper fractions	• Multiply the denominator by the whole number part. • Add the numerator to this. • Put the answer to this back over the denominator	$7\frac{1}{6} = \frac{6 \times 7 + 1}{6} = \frac{43}{6}$
Percentages			
18.	Percentage	Means 'out of 100'	
19.	Multiplier	A decimal you multiply by to represent a percentage	
		To use a multiplier to find a percentage, divide your percentage by 100, then multiply the amount by this value.	
20.	Percentage increase	Calculate the percentage and add onto the original	
		Or use a multiplier	$amount \times \frac{100 + \%\ increase}{100}$
21.	Percentage decrease	Calculate the percentage and subtract from the original	
		Or use a multiplier	$amount \times \frac{100 - \%\ increase}{100}$
22.	Percentage change	$\frac{Change}{Original} \times 100$	
23.	Express one number as a percentage of another	$\frac{Number\ 1}{Number\ 2} \times 100$	
24.	Reverse percentage	Use when asked to find the priginal amount after a percentage increase or decrease.	
		Original Value x Multiplier = New Value Original Value = $\frac{\text{New Value}}{\text{Multiplier}}$	
25.	Interest	A fee paid for borrowing money or money earnt through investing.	
26.	Simple interest	Interest that is calculated as a percentage of the original	I = Prt I – Interest P – Original amount r – interest rate t - time
27.	Compound interest	When interest is calculate on the original amount and any previous interest	

		OR $Original \times Multiplier^{time}$	$P\left(1+\frac{R}{100}\right)^{n}$ P – Original amount R – Interest rate n – the number of interest periods (e.g. yrs)
28.	Tax	A financial charge placed on sales or savings by the government e.g. VAT	
29.	Loss	Income minus all expenses, resulting in a negative value	
30.	Profit	Income minus all expenses, resulting in a positive value	
31.	Depreciation	A reduction in the value of a product over time	
32.	Annual	Means yearly	
33.	Per annum	Means per year	
34.	Salary	A fixed regular payment, often paid monthly	
FDP Conversions			
35.	Percentage to decimal	Divide by 100	
36.	Decimal to percentage	Multiply by 100	
37.	Fraction to percentage	Find an equivalent fraction with 100 as the denominator	
38.	Percentage to fraction	Write as a fraction over 100 then simplify	
39.	Fraction to decimal	Carry out division or convert to a percentage first	
40.	Decimal to fraction	Use place value to find the denominator and simplify or convert to a percentage first.	

Basics to memorise

41.	Fraction	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
	Decimal	**0.01**	**0.1**	**0.125**	**0.2**	**0.25**	**$0.\dot{3}$**	**0.5**	**$0.\dot{6}$**	**0.75**
	Percentage	1%	10%	12.5%	20%	25%	$33.\dot{3}\%$	50%	$66.\dot{7}\%$	75%

Equations

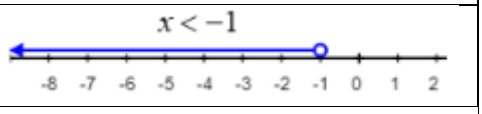
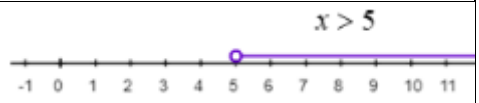
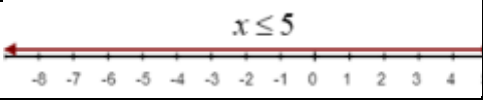
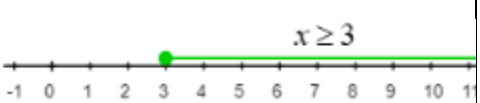
1.	Equation	Contains an 'equals' sign Has at least one variable	e.g. $3y - 3 = 12$
2.	Linear	Produces a straight line graph No indices above 1	
3.	Term	One part of an expression/equation/formula Can involve multiplying and dividing coefficients and variables Separated from other terms by addition and subtraction	e.g. $4c$ $\frac{w}{5}$
4.	Expression	One or a group of terms. Can include variables, constants, operators and grouping symbols. No 'equals' sign	e.g. $3y - 3$ $3y^2 + y^3$
5.	Formula	A special type of equation that shows the relationship between a set of variables	
6.	Identity	An equation that is true no matter what values are chosen, $\equiv$	e.g. $3y \equiv 2y - y$ for any value of y.
7.	Unknown	A letter representing a number	
8.	Solve	TO find the value of the unknown	
9.	Inverse operations	The operation used to reverse the original operation $+$ and $-$ are inverse / $\times$ and $\div$ are inverse Finding the square root is the inverse of finding the square of a number. Finding the cube root is the inverse of finding the cube of a number.	

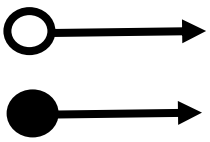
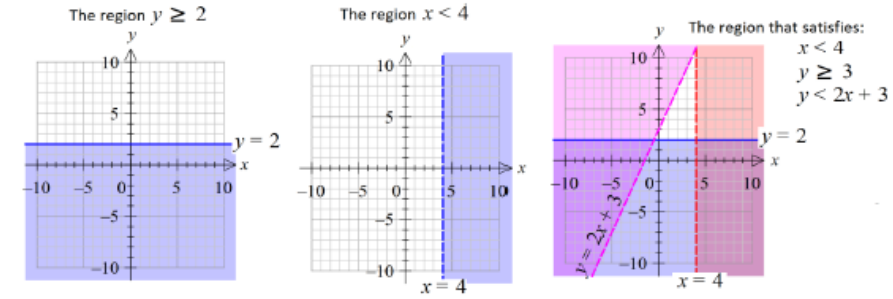
Solving equations

10.	To solve equations we need to use inverse operations	
11.	What ever you do to one side of the equals sign you must do the same to the other	
12.	One step	$x + 4 = 7$ (-4) (-4) $x = 11$ / $x - 5 = 12$ $(+5)$ $(+5)$ $x = 17$ / $3x = 18$ $(\div 3)$ $(\div 3)$ $x = 1$ / $\frac{x}{4} = 6$ $(\times 4)$ $(\times 4)$ $x = 24$
13.	Two step	Requires the use of two inverse operations $2x - 7 = 19$ $2x = 26$ $x = 13$

14.	With brackets	Expand the brackets first $5(2x + 1) = 35$ $10x + 5 = 35$ $10x = 30$ $x = 3$	OR if possible divide by the number outside of the bracket first $4(2x + 4) = 20$ $2x + 4 = 5$ $2x = 1$ $x = \frac{1}{2}$
15.	Unknowns on both sides	Start by eliminating the unknown from one of the signs.	$5x + 2 = 3x - 8$ $2x + 2 = -8$ $2x = -10$ $x = -5$
16.	With fractions	Eliminate any terms that are being added or subtracted separate from the fraction first. $\frac{f}{5} + 2 = 8$ $\frac{f}{5} = 6$ $f = 30$	If everything is part of the fraction then multiply by the denominator first. $\frac{f+2}{5} = 8$ $f + 2 = 40$ $f = 38$

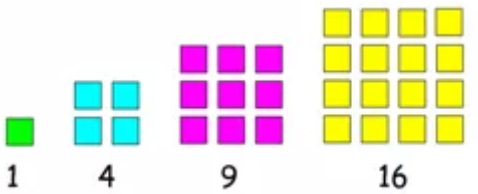
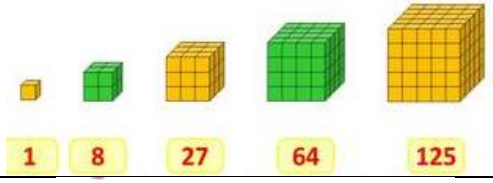
Inequalities

17.	Inequality	The relationship between two expressions that are not equal	
18.	=	Equal to	
19.	≠	Not equal to	
20.	<	Less than	
21.	>	Greater than	
22.	≤	Less than or equal to	
23.	≥	Greater than or equal to	
24.	Inclusive	Gives a finites rnage of solutions	e.g. $3 < x \leq 8$
25.	Exclusive	Gives an infinite range of solutions	e.g. $x > 5$ $-4 \leq x$
26.	Integer	A whole number that can be positive negative or zero	
27.	Solve	Inequalities are solved in the same way as solving equations	

		Only exception: if you multiply or divide by a negative number you must swap the sign e.g. less than to greater than	
28.	List integers solutions	Give the integers that satisfy the inequality	
		e.g. x > 6 integer solutions are 6, 7, 8....	
		e.g. -5 < x ≤ 5 integer solutions are -4, -3, -2, -1, 0, 1, 2, 3, 4, 5	
29.	Represent on a number line	An empty circle shows the value is not included	
		A shaded circle shows the value is included	
		An arrow shows that the solution continues to infinity	
30.	Inequalities on graphs	 The unwanted sections are shaded	
		Dashed lines are used to represent < or >	
		Solid line is used to represent $\leq$ *or* $\geq$	
Sequences			
31.	Sequence	An order pattern of numbers or diagrams	
32.	Term	One of the numbers or diagrams in a sequence	
33.	Term to term rule	The rule for moving from one term to the next in a sequence	
34.	Formula	A rule written to describe a realtionship between twp quantities	
35.	Arithmetic sequence	A sequence where the term to term rule is to addd or subtract the same amount each time	
36.	Quadratic sequence	A sequence where the term to term rule is changing by the same amount each time	
		The second difference is a constant amount.	
37.	Geometric sequence	A sequence where the term to term rule is to multiply by the same amount each time	
38.	Common ratio	The value a geometric sequence is multiplied by from one term to the next	
		Denoted by the letter *r*	

39.	Series	The sum of the terms in a sequence
40.	Position to term rule	The rule for finding any value of a sequence
41.	nth term rule for an arithmetic sequence	The rule to find any term in a sequence of numbers • Find the common difference between the terms • This becomes you coefficient of n (this is the times table the sequenc is linked to) • The number you need to add or subtract to get to the second term becomes the second term in the nth term rule 6, 10, 14, 18, 22 (+4 +4 +4 +4) The sequence increases by 4, so the nth term starts with 4n Now compare the sequence to the 4 times table 6, 10, 14, 18, 22 (+2 +2 +2 +2 +2) 4, 8, 12, 16, 20 Each term is 2 bigger than the 4 times table So the nth term is 4n + 2
42.	nth term for a geometric sequence	• Divide the second sequence by the first to find the common ratio, r • The nth term is ar^{n-1} where a is the first term and n is the term position in the sequence
43.	Finite	Has a final point
44.	Infinite	Carries on forever
45.	Ascending	Increases
46.	Descending	Decreases
47.	Linear function	An aruthmetic sequence that can be represented by a straight line graph

Special Sequences

48.	Square numbers	1, 4, 9, 16, 25, 36, 49, 64, 81, 100	
49.	Cube numbers	1, 8, 27, 64, 125	
50.	Triangular numbers	1, 3, 6, 10, 15, 21, 28	
51.	Fibonacci sequence	A sequence where each term is the sum of the two previous terms e.g. 1, 1, 2, 3, 5, 8, 13, 21...	