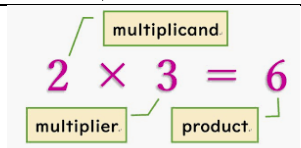
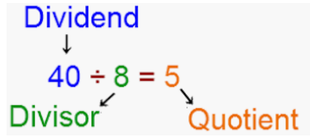
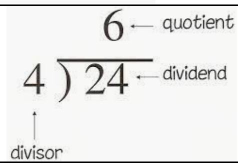
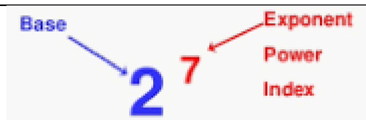
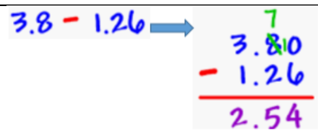
GCSE Mathematics
Higher
Unit 1

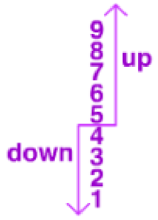
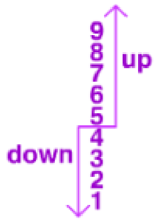
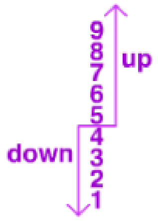
Definitions

Integer	A whole numbers and the negative equivalents.	
Positive	Greater than zero.	
Negative	Less than zero.	
Decimal	A number with digits after the decimal point.	
Operations	Symbols describing how to combine numbers. $\times \rightarrow$ Multiply, $\div \rightarrow$ Divide, $+ \rightarrow$ Add, $- \rightarrow$ Subtract,	
Multiplications terms	*Multiplicand:* The number being multiplied. *Multiplier*: The number that we are multiplying by. *Product*: The result of the multiplication operation.	
Division terms	*Dividend*: The number being divided. *Divisor:* The number we are dividing by. *Quotient:* The result of the division operation.	 
Inverse operations	The operation used to reverse the original operation.	$+$ and $-$ are inverses $\times$ and $\div$ are inverses Square and square root are inverses Cube and cube root are inverses
Order of Operations	The order in which operations should be done.	B — Brackets I — Indices DM — Division & Multiplication AS — Addition & Subtraction
$\neq$	Not equal to.	
Inclusive	Includes the first and last numbers given.	
Index Form	A number written as a base to the power of something.	
Prefix	The first part of a word, sometimes separated from the rest of the word by a hyphen.	
Standard Form	A number written in the form: $A \times 10^n$, where A is between 1 and 10.	
Scientific Notation	Another name for Standard Form.	
Surd	An method of writing non square or cube numbers as exact numbers in root form .	e.g. $\sqrt{4}$ is NOT a surd because $\sqrt{4} = 2$ $\sqrt{7}$ IS a surd because it is between 2 and 3
Fraction	Represents a proportion or part of a whole.	e.g. $\frac{4}{5}$
Numerator	The number or term on top of the fraction.	$\frac{Numerator}{Denominator}$
Denominator	The number or term on the bottom of the fraction.	
Rationalise the denominator	Eliminate a surd denominator in a fraction.	

1a. Calculations, checking and rounding (N2, N3, N5, N14, N15)

i)	Add & subtract decimals	Use the column method making sure making sure the decimal points are vertically aligned	

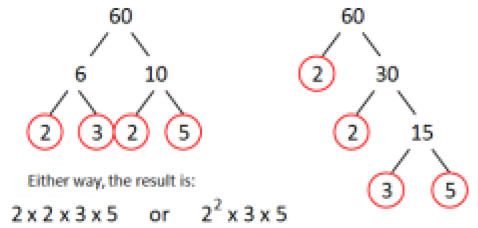
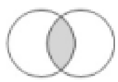
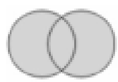
ii)	Multiply decimals	Multiply the integers and correct place value	Calculate: $\mathbf{4.32 \times 20.8}$ Use: $432 \times 208 = 89856$ So: $4.32 \times 20.8 = 89.856$ *2 dp* *1 dp* *3dp*
iii)	Divide decimals	*<u>Dividing a decimal by an integer</u>*: Use short division ensuring that a decimal point is placed vertically above the decimal point in the dividend.	3.7 4⟌14.8
		<u>Division with a decimal remainder</u>: add a decimal point and additional zero's after the dividend to allow you to continue the short division as above.	Calculate: $\mathbf{57 \div 8}$ Use: 0 7.1 2 5 8⟌5 7.^{1}0^{2}0^{4}0
		<u>Dividing by a decimal:</u> Multiply dividend and divisor by 10, 100 , 1000 so that the divisor becomes an integer then complete short division as above. <u>N.B. Do not place value after the calculation!</u>	Calculate: $\mathbf{6.488 \div 0.8}$ $\times 10$ $\times 10$ Use: $64.88 \div 8 = 8.11$ So: $6.488 \div 0.8 = \mathbf{8.11}$
iv)	Multiply any number between 0 and 1	Use the methods described above in: ii) Multiply decimals *N.B.* Value of the product will be smaller than the value of the multiplicand if the multiplier is between 0 and 1 and vice-versa.	$\mathbf{12} \times 0.2 = \mathbf{6}$ And: $0.2 \times \mathbf{12} = \mathbf{6}$
	Divide any number between 0 and 1	Use the methods described above in: iii) Divide decimals *N.B.* Value of the quotient will be greater than the value of the dividend if the divisor is between 0 and 1.	$\mathbf{12} \div 0.2 = \mathbf{60}$
v)	Use one calculation to find the answer to another	Given: $a \times b = c$ Then: $c \div b = a$ and $c \div a = b$ Adjust place value if necessary.	If: $\mathbf{19 \times 24 = 456}$ $456 \div 24 = 19$ $456 \div 19 = 24$ $1.9 \times 24 = 45.6$ $456 \div 190 = 2.4$ $19 \times 240 = 4560$
vi)	Use the product rule for counting: multiple groups	There are $\boldsymbol{n}$ different options available from group A and $\boldsymbol{m}$ different options available from group B. The number of possible combinations that can occur when choosing one option from Group A <u>and</u> one option from Group B is given by: $\boldsymbol{n \times m}$	e.g. A restaurant serves 4 different starter and 5 different main courses. How many combinations of start and main course could you choose? $\mathbf{4 \times 5 = 30}$
	Use the product rule for counting: one group with repeats	There are $\boldsymbol{n}$ possible options available from a single group and the same option can be selected multiple times. The number of possible combinations that can occur when choosing $\boldsymbol{m}$ options is given by: n^m	e.g. A combination lock has 3 wheels with the numbers 1 to 8 on each wheel. How many different combinations are possible? $\mathbf{8^3 = 512}$
	Use the product rule for counting: one group without repeats	There are n possible options from a single group and each options can be selected once only. The number of possible outcomes that can occur when choosing m options is given by: $n \times (n-1) \times (n-2) \times \ldots\ldots.. \times (n-m+1)$	e.g. 12 people run a marathon, how many combinations of gold, silver and bronze medal winners are there? $\mathbf{12 \times 11 \times 10 = 1320}$

vii)	Round to a given number of decimal places	• Count the number of decimal places you need. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. 36.3486343 $36.3\|486343$ To 1 d.p. is 36.3 $36.34\|86343$ To 2 d.p. is 36.35 $36.348\|6343$ To 3 d.p. is 36.349
ii)	Round a large number to a given number of significant figures	• Count the number of digits you need from the left. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down. • Replace remaining digits with zeros as place holders.		e.g. $324\,627\,938$ $3\|24627938$ To 1 s.f. is 300000000 $32\|4627938$ To 2 s.f. is 320000000 $324\|627938$ To 3 s.f. is 325000000
ix)	Round a small number to a given number of significant figures	• Zeros are not significant until after the first non-zero number. • Find the first non-zero and count the number of digits you need from there. • Look at the number to the right of that digit to decide if it should round up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. 0.0034792 $0.003\|4792$ To 1 s.f. is 0.003 $0.0034\|792$ To 2 s.f. is 0.0035 $0.00347\|92$ To 3 s.f. is 0.00348
x)	Estimating	• Round each number to 1 significant figure before doing any calculations. • It is acceptable to round one or more numbers in the calculation to a greater accuracy than 1 sig. fig. if this makes the calculation easier. • DO NOT round the answer!		e.g. Estimate: $\frac{3.91 \times 8789.8}{620.9 \times 0.492}$ $\frac{3.91 \times 8789.8}{620.9 \times 0.492} \approx \frac{4 \times 9000}{600 \times 0.5}$ $\approx \frac{3600}{300}$ ≈ 120

1b. Indices, roots, reciprocals and hierarchy of operations (N2, N3, N6, N7, N14)

X i)	Use index notation for positive powers of 10	• Count how many zero's there are after the 1 and write 10 to the power of this number. • Write a 1 followed by the same number of zero's as the power 10 is raised to.	e.g. $10\,000\,000 = 10^7$ e.g. $10^2 = 100$
ii)	Use index notation for negative powers of 10	• Count how many zero's there are in front of the 1 and write 10 to the power of the negative of this number. • Use the positive of the power 10 is raised to and write a 1 with this number of zero's in front with a decimal point after the first.	e.g. $0.000\,000\,1 = 10^{-7}$ e.g. $10^{-2} = 0.01$

iii)	Recognise common powers	Recall that the positive power of a number tells us how many times to use that number in a multiplication.		e.g. $3^4 = 3 \times 3 \times 3 \times 3$ e.g. $7^2 = 7 \times 7$
	Powers of 2	$2^1 = \mathbf{2}, 2^2 = \mathbf{4}, 2^3 = \mathbf{8}, 2^4 = \mathbf{16}, 2^5 = \mathbf{32}, 2^6 = \mathbf{64}, 2^7 = \mathbf{128}, 2^8 = \mathbf{256}. 2^9 = \mathbf{512}, 2^{10} = \mathbf{1024}$		
	Powers of 3	$3^1 = \mathbf{3}, 3^2 = \mathbf{9}, 3^3 = \mathbf{27}, 3^4 = \mathbf{81}, 3^5 = \mathbf{243}$		
	Powers of 4	$4^1 = \mathbf{4}, 4^2 = \mathbf{16}, 4^3 = \mathbf{64}, 4^4 = \mathbf{256}, 4^5 = \mathbf{1024}$		
	Powers of 5	$5^1 = \mathbf{5}, 5^2 = \mathbf{25}, 5^3 = \mathbf{125}, 5^4 = \mathbf{625}$		
iv)	Estimate roots of any given positive number	• Identify the square (or cube) numbers immediately above and below the number we are trying to find the square (or cube) root of. • The desired root must lie between the integer roots of the square numbers immediately above and below.		e.g. Between which two integers does $\sqrt{42}$ lie? • Next square number is $\mathbf{49}$. • Previous square number is $\mathbf{36}$. • $\sqrt{\mathbf{36}} = \mathbf{6}, \sqrt{\mathbf{49}} = \mathbf{7}$ • So: $\sqrt{\mathbf{42}}$ lies between : $\mathbf{6} \;\&\; \mathbf{7}$
v)	Find the value of calculations involving positive indices	Recall that a positive power of a number tells us how many times to use that number in a multiplication.		e.g. $3^4 = 3 \times 3 \times 3 \times 3$ e.g. $7^2 = 7 \times 7$
	Find the value of calculations involving negative indices	To calculate a negative power: • Calculate the equivalent positive power. • Then take the reciprocal.	$a^{-n} = \frac{1}{a^n}$	e.g. Calculate $\mathbf{4^{-3}}$. • $4^3 = 64$ • $4^{-3} = \mathbf{\frac{1}{64}}$
	Find the value of calculations involving fractional indices	The denominator of the fractional power gives the type of root to evaluate.	$a^{\frac{1}{n}} = \sqrt[n]{a}$	e.g. $\mathbf{64^{\frac{1}{2}}} = \sqrt{\mathbf{64}} = \mathbf{8}$ e.g. $\mathbf{125^{\frac{1}{3}}} = \sqrt[3]{\mathbf{125}} = \mathbf{5}$
vi)	Use powers of 0 and 1	Anything to the power of $0 = 1$	$a^0 = 1$	e.g. $\mathbf{5^0} = \mathbf{1}$
		Anything to the power $1 =$ itself	$a^1 = a$	e.g. $\mathbf{5^1} = \mathbf{5}$
vii)	Use index laws to simplify or evaluate numerical expressions	*Multiplication* • Add the powers	$a^m \times a^n = a^{m+n}$	e.g. $\mathbf{2^2} \times \mathbf{2^3} = \mathbf{2^5}\ (= \mathbf{32})$
		Division • Subtract the powers	$a^m \div a^n = a^{m-n}$	e.g. $\mathbf{3^9} \div \mathbf{3^4} = \mathbf{3^5}\ (= \mathbf{243})$
		Brackets • Multiply the powers	$(a^m)^n = a^{mn}$	e.g. $\mathbf{(7^4)^3} = \mathbf{7^{12}}$

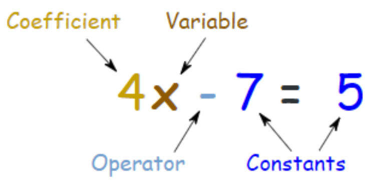
1c. Factors, multiples and primes (N3, N4)			
i)	Factors	A factor is a number that divides into another number	e.g. factors of 6: 1, 2, 3 and 6
ii)	Multiples	A multiple is a number from the times tables	e.g. multiples of 4: 4, 8, 12, 16, 20, ………
iii)	Prime number	A prime number is a number with exactly 2 factors	
		2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97	
iv)	Product	The answer when two or more numbers are multiplied together.	e.g. Product of 3 & 7: $3 \times 7 = \mathbf{21}$
v)	Prime factor decomposition	Writing a number as a *product of its prime factors*	
vi)	Highest common factor (HCF)	The highest number that divides exactly into two or more numbers.	 e.g. The HCF of 12 & 8: $\mathbf{4}$
vii)	Lowest common multiple (LCM)	The smallest positive number that is a multiple of two or more numbers.	 e.g. The LCM of 12 & 8: $\mathbf{24}$
1d. Standard form (N9)			
i)	Convert a small number to standard form	• Count the number of zero's in front of the first significant figure (including the one in front of the decimal point). • The power of ten is negative followed by this number.	e.g. $\mathbf{0.00000037}$ $= \mathbf{3.7 \times 10^{-7}}$
ii)	Convert a large number into standard form	• Count the number of place value position there are after the first significant figure. • The power of ten is positive followed by this number.	e.g. $\mathbf{147\ 100\ 000\ 000}$ $= \mathbf{1.47 \times 10^{11}}$
iii)	Converting to a small ordinary number	• Look at the digit after the negative in the power of 10. • Write this may zero's in front of the first sig. fig. • Reposition the decimal place between the first and second zero.	e.g. $\mathbf{2.4 \times 10^{-6}}$ $= \mathbf{0.0000024}$
iv)	Adding or subtracting numbers in standard form	• Convert the numbers to ordinary numbers. • Add. • Convert the sum to standard form.	e.g. $(\mathbf{2.3} \times \mathbf{10^4}) + (\mathbf{6.4} \times \mathbf{10^3})$ $= 23000 + 6400$ $= 29400$ $= \mathbf{2.94} \times \mathbf{10^4}$

v)	Multiplying numbers in standard form	• Multiply the numbers between one and 10 at the front. • Use index law for multiplication for the powers of 10. • If necessary increase the power of ten by one to ensure the initial number is between 1 and 10.	e.g. $(\mathbf{4.5} \times \mathbf{10^3}) \times (\mathbf{3} \times \mathbf{10^5})$ $= 13.5 \times 10^{3+5}$ $= 13.5 \times 10^8$ $= \mathbf{1.35 \times 10^9}$
vi)	Dividing numbers in standard form	• Divide the numbers between one and 10 at the front. • Use index law for division for the powers of 10. • If necessary decrease the power of ten by one to ensure the initial number is between 1 and 10.	e.g. $(\mathbf{2.5} \times \mathbf{10^{11}}) \div (\mathbf{5} \times \mathbf{10^{13}})$ $= 0.5 \times 10^{-2}$ $= \mathbf{5 \times 10^{-3}}$
1d. Surds (N8)			
i)	Multiply	$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\sqrt{a} \times \sqrt{a} = a$	e.g. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ and $\sqrt{3} \times \sqrt{3} = 3$
ii)	Divide	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	e.g. $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$
iii)	Add and subtract	$\sqrt{a} + \sqrt{b}$ cannot simplify	e.g. $\sqrt{3} + \sqrt{2} = \sqrt{3} + \sqrt{2}$
		But $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$	e.g. $5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$
iv)	Simplify	$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$	e.g. $\sqrt{50} + \sqrt{18} = 5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$
v)	Rationalise the denominator	Multiply numerator and denominator (use equivalent fractions) by whatever will result in the denominator simplifying to an integer.	e.g. $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{7}{\sqrt{7}}$ e.g. $\frac{1}{5+\sqrt{2}} = \frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{5-\sqrt{2}}{3}$

GCSE Mathematics
Higher
Unit 2

Algebra: the basics

Definitions

1.	Variable	A letter representing a varying or unknown quantity.	
2.	Coefficient	A number which multiplies a variable e.g. 4 is the coefficient in 4a	
3.	Term	One part of an expression/equation/formula Can involve multiplying and dividing coefficients and variables Separated from other terms by addition and subtraction	e.g. 4c $\frac{w}{5}$
4.	Like terms	Terms that have the same variable but have different coefficients	e.g. c + 4c are like terms c^2 and c^3 are not like terms
5.	Constant	A fixed value. A number on its own or sometimes a letter such as a, b or c to represent a fixed number.	
6.	Expression	One or a group of terms. Can include variables, constants, operators and grouping symbols. No 'equals' sign	e.g. 3y -3 $3y^2 + y^3$
7.	Equation	Contains an 'equals' sign, = Has at least one variable	e.g. 3y – 3 = 12
8.	Formula	A special type of equation that shows the relationship between a set of variables	
9.	Formulae	Plural of 'formula'	
10.	Identity	An equation that is true no matter what values are chosen, ≡	e.g. 3y ≡ 2y – y for any value of y.
11.	Subject	The variable on its own on one side of the equals sign.	
12.	Substitute	Replace a variable with a number.	$a = 3, b = 2\ and\ c = 5.$ **Find:** 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$
13.	Simplify	Minimising the size of an expression	

14.	Factorise	Splitting an expression into a product of factors	
15.	Expand	Removing brackets by using multiplication	
16.	Solve	Find the value of an unknown	

Algebraic Notation

17.	Adding like terms	Add the coefficients	$b + 2b = 3b$
18.	Subtracting like terms	Subtract the coefficients	$5b - 4b = b$
19.	Multiplying like terms	If the base is the same, add the powers	$b \times b = b^2$
20.	Dividing terms	If the base is the same, subtract the powers	$b^5 \div b^2 = b^3$
21.	Adding different terms	Cannot combine if the terms are different.	$b + 2c = b + 2c$
22.	Subtracting different terms	Cannot combine if the terms are different.	$3c - 4 = 3c - 4$
23.	Multiplying different terms	Combine with no '×' sign	$d \times e = de$
24.	Multiplying different terms with coefficients	Combine with no '×' sign, multiply the coefficients	$2d \times 3e = d6e$
25.	Dividing different terms	Write as fractions with no '÷' sign	$3d \div e = \frac{3d}{e}$
26.	Dividing different terms with coefficients	Write as fractions with no '÷' sign, simplify the coefficients where possible.	$14d \div 7e = \frac{2d}{e}$

Expanding (single brackets)

27.	Multiply all the terms inside the bracket, by the term on the outside.		
28.	$3(a+4) = 3a+12$	$\begin{array}{c\|c\|c} \times & 2x & -3 \\ \hline 2x & 4x^2 & -6x \end{array}$	$4x^2 - 6x$

Factorising (single brackets)

29.	• Find the highest common factor of the terms • This goes outside the bracket • Divide each term by the factor to get the new terms inside the bracket • Always check by expanding your bracket	$2x + 4y$ $5x^2y - 10xy$	$2(x + 2y)$ $5xy(x - 2)$

Expressions

30.	Linear	Can be represented by a straight line No indices above 1	e.g. $2x + 2$
31.	Quadratic	An expression where the highest index is 2	e.g. $2x^2 + 2x + 2$

Expanding double brackets

32.	Everything in the first bracket must be multiplied by everything in the second	
33.	**Grid method** $(x+4)(x+7)$ $\begin{array}{c\|c\|c} \times & x & +4 \\ \hline x & x^2 & 4x \\ \hline +7 & 7x & 28 \end{array}$ $= x^2 + \underline{4x + 7x} + 28$ $= \underline{x^2 + 11x + 28}$	**FOIL method** FIRST : $(x+3)(x-4)$ gives $x \times x = x^2$ OUTER : $(x+3)(x-4)$ gives $x \times (-4) = -4x$ INNER : $(x+3)(x-4)$ gives $3 \times x = 3x$ LAST : $(x+3)(x-4)$ gives $3 \times (-4) = -12$

Factorising a quadratic expression

34.	Factorising a quadratic in the form of $ax^2 + bx + c$	Multiply to 5 Factorise $x^2 + 5x + 6$ ← Add to 6 2 and 3 add to 5 2 and 3 multiply to 6 $(x+2)(x+3)$ Check: $(x+2)(x+3) = x^2 + 5x + 6$
35.	Difference of two squares	A special type of quadratic which only has two terms. One term is subtracted from the other $x^2 - 25 = x^2 - 5^2 = (x+5)(x-5)$ $y^2 - 49 = y^2 - 7^2 = (y+7)(y-7)$ $a^2 - 16 = a^2 - 4^2 = (a+4)(a-4)$

Equations

36.	To solve equations we need to use inverse operations
37.	What ever you do to one side of the equals sign you must do the same to the other

38.	One step	$x + 4 = 7$ $(-4) \quad (-4)$ $x = 11$ $x - 5 = 12$ $(+5) \quad (+5)$ $x = 17$ $3x = 18$ $(\div 3) \quad (\div 3)$ $x = 1$ $\frac{x}{4} = 6$ $(\times 4) \quad (\times 4)$ $x = 24$	
39.	Two step	Requires the use of two inverse operations	$2x - 7 = 19$ $2x = 26$ $x = 13$
40.	With brackets	Expand the brackets first $5(2x + 1) = 35$ $10x + 5 = 35$ $10x = 30$ $x = 3$	OR if possible divide by the number outside of the bracket first $4(2x + 4) = 20$ $2x + 4 = 5$ $2x = 1$ $x = \frac{1}{2}$
41.	Unknowns on both sides	Start by eliminating the unknown from one of the signs.	$5x + 2 = 3x - 8$ $2x + 2 = -8$ $2x = -10$ $x = -5$
42.	With fractions	Eliminate any terms that are being added or subtracted separate from the fraction first. $\frac{f}{5} + 2 = 8$ $\frac{f}{5} = 6$ $f = 30$	If everything is part of the fraction then multiply by the denominator first. $\frac{f + 2}{5} = 8$ $f + 2 = 40$ $f = 38$

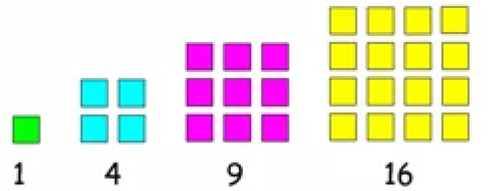
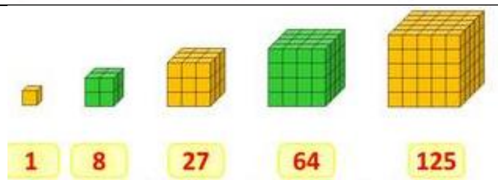
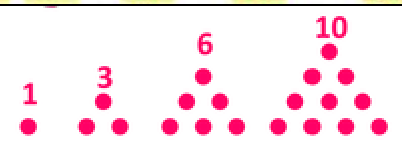
Changing the subject of a formula (rearranging)

43.	Always use inverse operations to isolate the term you have been asked to make the subject		
	If the letter you want as the subject appears twice you will need to factorise		
	Make u the subject: $v = u + at$ $(-\boldsymbol{at})$ $v - at = u$ So $u = v - at$	Make u the subject: $v^2 = u^2 + 2as$ $(-\boldsymbol{2as})$ $v^2 - 2as = u^2$ $(\sqrt{\ })$ $\sqrt{v^2 - 2as} = u$ So $u = \sqrt{v^2 - 2as}$	Make m the subject: $I = mv - mu$ $(\boldsymbol{Factorise})$ $I = m(v - u)$ $(\div (\boldsymbol{v - u}))$ $\frac{I}{v - u} = m$ So $m = \frac{I}{v - u}$

Iteration		
44.	Iteration	The act of repeating a process to generate a sequence of outcomes or with the aim of of appraoching a desired result e.g. finding a solution to an equation
45.	Iterative sequence	The relationship between consecutive terms
46.	Roots	Solutions to an equation
47.	Change of sign	Two values with a root between them
Sequences		
48.	Sequence	An order pattern of numbers or diagrams
49.	Term	One of the numbers or diagrams in a sequence
50.	Term to term rule	The rule for moving from one term to the next in a sequence
51.	Formula	A rule written to describe a realtionship between twp quantities
52.	Arithmetic sequence	A sequence where the term to term rule is to addd or subtract the same amount each time
53.	Quadratic sequence	A sequence where the term to term rule is changing by the same amount each time The second difference is a constant amount.
54.	Geometric sequence	A sequence where the term to term rule is to multiply by the same amount each time
55.	Common ratio	The value a geometric sequence is multiplied by from one term to the next Denoted by the letter r
56.	Series	The sum of the terms in a sequence
57.	Position to term rule	The rule for finding any value of a sequence
58.	nth term rule for an arithmetic sequence	The rule to find any term in a sequence of numbers • Find the common difference between the terms • This becomes you coefficient of n (this is the times table the sequenc is linked to) • The number you need to add or subtract to get to the second term becomes the second term in the nth term rule 6, 10, 14, 18, 22 +4 +4 +4 +4 The sequence increases by 4, so the nth term starts with 4n Now compare the sequence to the 4 times table 6, 10, 14, 18, 22 +2 +2 +2 +2 +2 4, 8, 12, 16, 20 Each term is 2 bigger than the 4 times table So the nth term is 4n + 2
59.	Nth term for a quadratic sequence	• Find the first difference • Find the second difference • Halve the second difference and multiply by n^2 to gain a new sequence of an^2 • Generate the first few term sof this seuence then subtract from the original sequence

		• Find the nth term of the remianing sequence $bn + c$ • The entire nth term is then $an^2 + bn + c$
60.	nth term for a geometric sequence	• Divide the second sequence by the first to find the common ratio, *r* • The nth term is ar^{n-1} where *a* is the first term and *n* is the term position in the sequence
61.	Finite	Has a final point
62.	Infinite	Carries on forever
63.	Ascending	Increases
64.	Descending	Decreases
65.	Linear function	An aruthmetic sequence that can be represented by a straight line graph

Special Sequences

66.	Square numbers	1, 4, 9, 16, 25, 36, 49, 64, 81, 100	
67.	Cube numbers	1, 8, 27, 64, 125	
68.	Triangular numbers	1, 3, 6, 10, 15, 21, 28	
69.	Fibonacci sequence	A sequence where each term is the sum of the two previous terms	
		e.g. 1, 1, 2, 3, 5, 8, 13, 21...	

GCSE Mathematics Higher Unit 3

Definitions

1.	Qualitative Data	Non-numerical data	i.e. Colour of car
2.	Quantitative Data	Numerical data	i.e. House number
3.	Discrete Data	Numerical data that CANNOT be shown in decimals	i.e. Number of children in a class
4.	Continuous Data	Numerical data that CAN be shown in decimals	i.e. The heights of children in a class
5.	Grouped Data	Numerical data given in intervals	i.e. Year group ranges: Year 7-9 Year 10-11 Year 12-13

Averages

6.	Measure of location	A single value that describes a position in a data set
7.	Measure of central tendency	A single value that describes the centre of the data
8.	Measure of spread	A measure of how spread out the data is
		Also known as 'measures or dispersion' or 'measures of variation'
		Two simple measures of spread are range and interquartile range (IQR)
9.	Mode (modal class)	The value that occurs most often
10.	Range	The difference between the largest and smallest values in the data set
11.	Median	The middle value when the data values are put in ascending order
12.	Mean	Found by adding all number sin the data set and dividing by the number of values in the set
		Can be calculate using the formula $\bar{x} = \frac{\Sigma x}{n}$ — Where: $\bar{x}$ is the mean, Σx is the sum of the data values, n is the number of data values
		Mean from a frequency table $$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$ Where Σfx is the sum of the products of data values and their frequencies and Σf is the sum of the frequencies

Advantages and disadvantages of averages

	Average	Advantages	Disadvantages
13.	*Mean*	Every value makes a difference	Affected by extreme values
	Median	Not affected by extreme values	May not change even if a data value changes
	Mode	Easy to find; not affected by extreme values; can be used for non-numerical data	There may not be a mode

Averages from frequency tables

14.	Modal class	The class with the highest frequency
15.	Median	If the total frequency is n, then the median lies in the class with the $\frac{n+1}{2}$th value in it.
16.	Mean from a frequency table Times → Add ↓↓ Divide ←	(see table below)
17.	Estimated mean from a grouped frequency table Times → Add ↓↓ Divide ←	(see table below)
18.	Estimate of range from grouped frequency table	The maxiumum possible value minus the smallest possible value.

16. No of make-up items in handbags

No of Items x	Freq f	f x x
1	7	1 x 7 = 7
2	2	2 x 2 = 4
3	1	3 x 1 = 3
4	4	4 x 4 = 16
5	2	5 x 2 = 10
	16	40

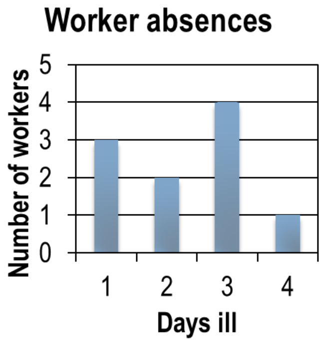
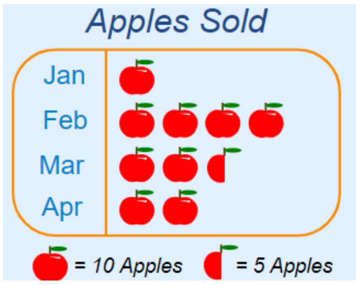
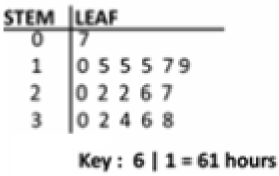
$$\text{Mean} = \frac{40}{16} = 2.5$$

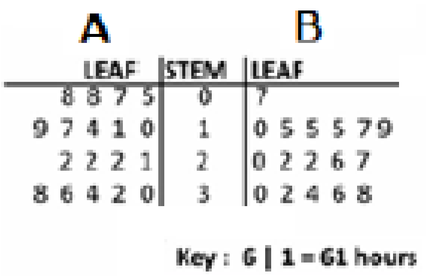
17.

Class Interval	*Mid-point*	*Frequency*	*Mid-point × Frequency*
$140 \leq h < 150$	145	6	$145 \times 6 = 870$
$150 \leq h < 160$	155	16	$155 \times 16 = 2480$
$160 \leq h < 170$	165	21	$165 \times 21 = 3465$
$170 \leq h < 180$	175	8	$175 \times 8 = 1400$
	Totals	**51**	**8215**

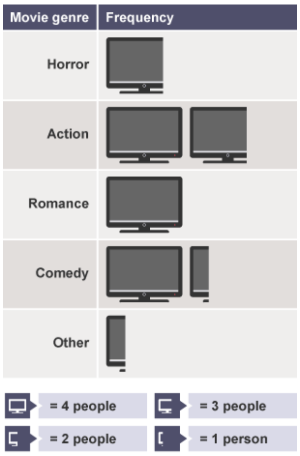
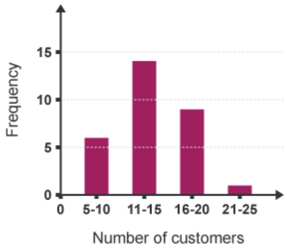
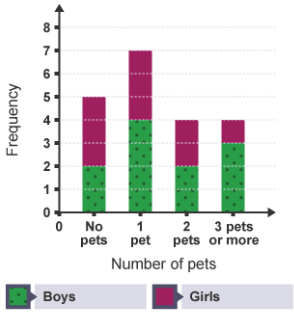
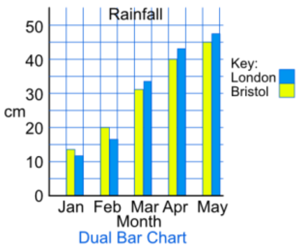
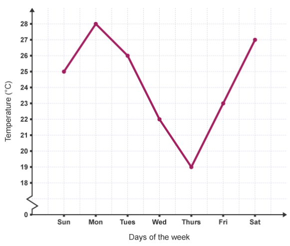
Mean = 8215 ÷ 51
=161.07843...
= 161.08 (2dp)

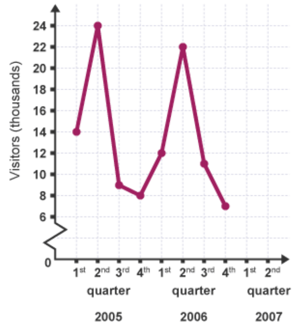
Averages from charts/graphs

<table>
<tr><td>19.</td><td>Bar chart</td><td>A chart to display discrete data where the height of the bar shows the frequency.

</td><td>Mean: 23 ÷ 10 = 2.3
Median: 2.5
Mode : 3
Range: 4-1 = 3</td></tr>
<tr><td>20.</td><td>Pictogram</td><td>A chart that uses pictures to represent quantities. Must include a key.

</td><td>Mean: 95÷4 = 23.75
Median: 22.5
Range: 30</td></tr>
<tr><td>21.</td><td>Stem and leaf diagram</td><td>

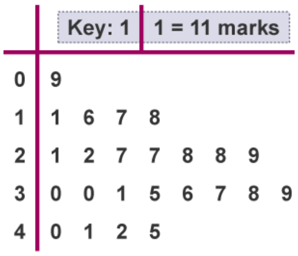
A diagram that shows groups of data arranged by place value. ‘Leaves’ should be in order. Must have a key.</td><td>Mean: 385÷17 = 22.6
Median: 22
Mode: 15
Range: 38-7 = 31</td></tr>
<tr><td>22.</td><td>Back to back stem and leaf</td><td>Compares two sets of results. Must have a key.

</td><td>Set A
Mean: 356÷18 = 19.8
Median: 20
Mode: 22
Range: 38-5 = 33

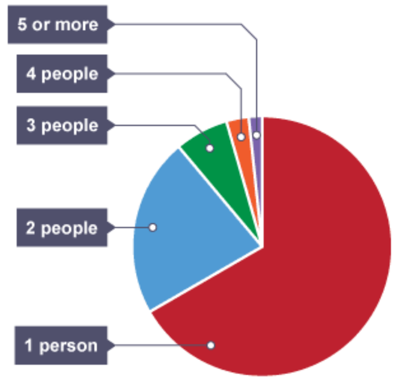
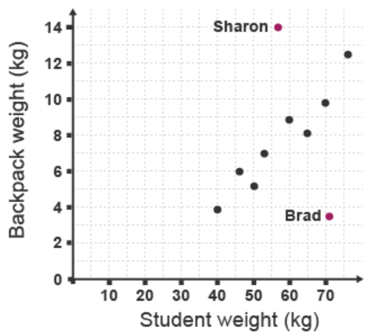
Set B
Mean: 385÷17 = 22.6
Median: 22
Mode: 15
Range: 38-7 = 31</td></tr>
<tr><td colspan="4"><h2>Representing data</h2></td></tr>
<tr><td>23.</td><td>Two-Way Tables</td><td><table><tr><td></td><td>Boys</td><td>Girls</td><td>TOTAL</td></tr><tr><td>Pet</td><td>9</td><td>4</td><td>13</td></tr><tr><td>No Pet</td><td>2</td><td>5</td><td>7</td></tr><tr><td>TOTAL</td><td>11</td><td>9</td><td>20</td></tr></table></td><td>Two-way tables are a way of sorting data with two categories.</td></tr>
</table>

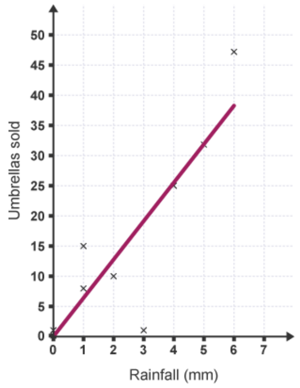
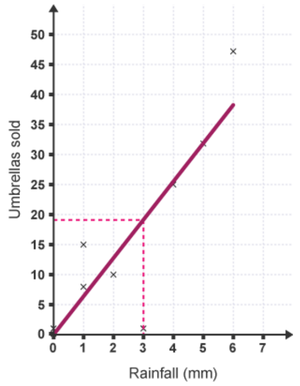
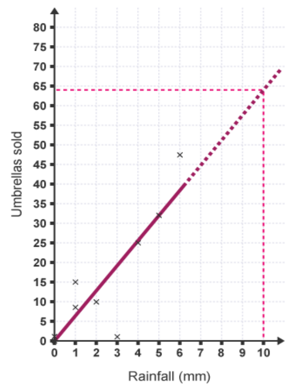
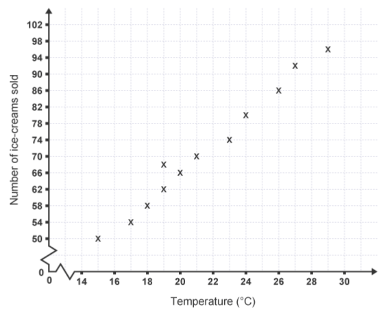
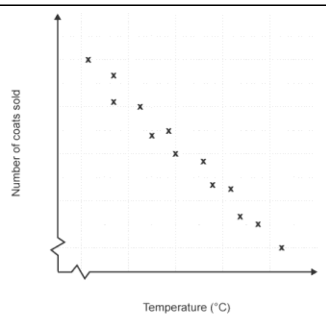
24.	Pictograms		Used to show frequencies Pictures and images used to represent frequency A key at the bottom helps you interpret the diagram
25.	Bar Charts		Frequency on the vertical axis, and categories along the horizontal axis. Used to compare frequencies
26.	Composite Bar Chart		Frequency on the vertical axis, and categories along the horizontal axis. Two shades used to show difference in proportion between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
27.	Comparative Bar Chart		Frequency on the vertical axis, and categories along the horizontal axis. Bars are next to each other and used to show difference in frequency between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
28.	Line Graph		A line graph is used to show a change or relationship between two variables. Once the points are plotted, they are joined with straight lines.

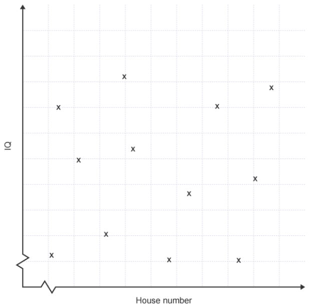
<table>
<tr><td>29.</td><td>Time-Series</td><td>
</td><td>A time-series graph plots frequencies (vertical) axis against time (horizontal).

It is used to spot trends over time.

Time could be: weeks, months, quarters (3 months), years</td></tr>
<tr><td>30.</td><td>Stem & Leaf Diagrams:</td><td>
</td><td>A stem and leaf diagram shows numbers in a table format. It can be a useful way to organise data to find the median, mode and range of a set of data.

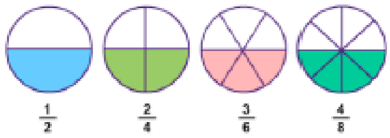
Only one digit is allowed to be a ‘leaf’
There should be a key to help you interpret the diagram</td></tr>
<tr><td>31.</td><td>Pie Charts</td><td colspan="2">A pie chart is a chart represented by a circle. It shows the proportion of each group at a glance.
<table>
<tr><th>People travelling in a vehicle</th><th>Frequency</th><th>Calculation</th><th>Angle</th></tr>
<tr><td>1 person</td><td>120</td><td>$\frac{120}{180} \times 360°$</td><td>240°</td></tr>
<tr><td>2 people</td><td>40</td><td>$\frac{40}{180} \times 360°$</td><td>80°</td></tr>
<tr><td>3 people</td><td>13</td><td>$\frac{13}{180} \times 360°$</td><td>24°</td></tr>
<tr><td>4 people</td><td>5</td><td>$\frac{5}{180} \times 360°$</td><td>10°</td></tr>
<tr><td>5 or more</td><td>2</td><td>$\frac{2}{180} \times 360°$</td><td>4°</td></tr>
<tr><td>Total</td><td>180</td><td></td><td></td></tr>
</table>

</td></tr>
<tr><td colspan="4">Scatter Graphs</td></tr>
<tr><td>32.</td><td>Outliers</td><td>
</td><td>Outliers don’t follow the trend</td></tr>
</table>

33.	Line of Best Fit		A sensible straight line that goes as centrally as possible through the points plotted. It should also follow the same steepness of the crosses.	
34.	Interpolate		Using a line of best fit to estimate data <u>WITHIN</u> our range <u>For example</u>: To estimate how many umbrellas are sold with 3mm rain. • Find where 3 mm of rainfall is on the graph. • Draw a line by going across from 3 mm and then down.	
35.	Extrapolate		Continuing a line of best fit to estimate data <u>BEYOND</u> our range (not as reliable as interpolation) <u>For example</u>: To estimate how many umbrellas are sold with 10mm rain. • Continue the line of best fit. • Find where 10mm of rainfall is on the graph. • Draw a line by going across from 10mm and then down.	
36.	Positive Correlation		BOTH variables increase with each other	i.e. Ice creams sold vs Temperature
37.	Negative Correlation		ONE variable increases as the other decreases	i.e. Coats sold vs temperature

38.	No Correlation		NO relationship between variables	i.e. IQ and House Number
39.	Causation	If one variable causes a change in the other. • i.e. an increase temperature WILL cause an increase ice cream sales • i.e. the number of bee stings WILL NOT cause an increase in ice cream sales (although both will increase in hot weather)		

GCSE Mathematics
Higher
Unit 4

Fractions

1.	Fraction	Part of a whole	
2.	Numerator	The number on the top of the fraction	$\frac{numerator}{denominator}$
3.	Denominator	The number on the bottom of the fraction	
4.	Equivalent fractions	Fractions that have the same value but look different.	
5.	Improper fraction	A fraction where the numerator is larger than the denominator.	e.g. $\frac{4}{3}$
6.	Mixed number	A number made from integer and fraction parts.	e.g. $2\frac{2}{3}$
7.	Unit fraction	A fraction that has a numerator of 1	
8.	Reciprocal	The reciprocal of a number is 1 divided by the number.	e.g. the reciprocal of 3 is $\frac{1}{3}$
		Dividing by a number is the same as multiplying by its reciprocal	e.g. $\times$ by $\frac{1}{3}$ is the same as $\div$ by 3

Fractions - processes

9.	Simplifying fractions	Divide the numerator and denominator by the HCF.	$\frac{24}{30} = \frac{4}{5}$
10.	Finding equivalent fractions	Multiply the numerator and denominator by the same number	$\frac{4 \times 2 = 8}{8 \times 2 = 16}$
11.	Comparing fractions	Write them with a common denominator	
12.	Fraction of an amount	Amount divided by the denominator then multiplied by the numerator	e.g. $\frac{5}{7}$ of 42 $42 \div 7 \times 5 = 30$
13.	Multiply fractions	Multiply the numerators and multiply the denominators	$\frac{6}{7} \times \frac{4}{5} = \frac{6 \times 4}{7 \times 5} = \frac{24}{35}$
14.	Divide fractions	• Flip the second fraction (find the reciprocal). • Change the divide to multiply. • Multiply the fractions.	$\frac{4}{7} \div \frac{5}{6} = \frac{4}{7} \times \frac{6}{5} = \frac{4 \times 6}{7 \times 5} = \frac{24}{35}$
15.	Add or subtract fractions	• Write as fractions with a common denominator. • Add or subtract the numerators	$\frac{2}{8} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$

16.	Convert improper fractions to mixed numbers	• Divide the numerator by the denominator • The answer gives the whole number part. • The remainder becomes the numerator of the fraction part with the same denominator.	$\frac{43}{6} = 7\frac{1}{6}$
17.	Convert mixed numbers to improper fractions	• Multiply the denominator by the whole number part. • Add the numerator to this. • Put the answer to this back over the denominator	$7\frac{1}{6} = \frac{6 \times 7 + 1}{6} = \frac{43}{6}$
18.	Adding and subtracting mixed numbers	• Convert mixed numbers to improper fractions • Transform both fractions so they have the same denominator • Add or subtract the numerators Convert back to mixed number if applicable	
19.	Multiplying mixed numbers	• Convert mixed numbers to improper fractions • Multiply numerators and multiply the denominators Convert back to mixed number if applicable	
20.	Dividing mixed numbers	• Convert mixed numbers to improper fractions • Flip the second fraction (find the reciprocal) • Change the divide sign to a multiply • Multiply the fractions Convert back to mixed number if applicable	
Percentages			
21.	Percentage	Means 'out of 100'	
22.	Multiplier	A decimal you multiply by to represent a percentage	
		To use a multiplier to find a percentage, divide your percentage by 100, then multiply the amount by this value.	
23.	Percentage increase	Calculate the percentage and add onto the original	
		Or use a multiplier	$amount \times \frac{100 + \%\ increase}{100}$
24.	Percentage decrease	Calculate the percentage and subtract from the original	
		Or use a multiplier	$amount \times \frac{100 - \ \%\ increase}{100}$
25.	Percentage change	$\frac{Change}{Original} \times 100$	
26.	Express one number as a percentage of another	$\frac{Number\ 1}{Number\ 2} \times 100$	

27.	Reverse percentage	Use when asked to find the priginal amount after a percentage increase or decrease. Original Value x Multiplier = New Value $\text{Original Value} = \dfrac{\text{New Value}}{\text{Multiplier}}$	
28.	Interest	A fee paid for borrowing money or money earnt through investing.	
29.	Simple interest	Interest that is calculated as a percentage of the original	I = Prt I – Interest P – Original amount r – interest rate t - time
30.	Compound interest	When interest is calculate on the original amount and any previous interest OR $Original \times Multiplier^{time}$	$P\left(1+\frac{R}{100}\right)^n$ P – Original amount R – Interest rate n – the number of interest periods (e.g. yrs)
31.	Tax	A financial charge placed on sales or savings by the government e.g. VAT	
32.	Loss	Income minus all expenses, resulting in a negative value	
33.	Profit	Income minus all expenses, resulting in a positive value	
34.	Depreciation	A reduction in the value of a product over time	
35.	Annual	Means yearly	
36.	Per annum	Means per year	
37.	Salary	A fixed regular payment, often paid monthly	
FDP Conversions			
38.	Percentage to decimal	Divide by 100	
39.	Decimal to percentage	Multiply by 100	
40.	Fraction to percentage	Find an equivalent fraction with 100 as the denominator	
41.	Percentage to fraction	Write as a fraction over 100 then simplify	
42.	Fraction to decimal	Carry out division or convert to a percentage first	
43.	Decimal to fraction	Use place value to find the denominator and simplify or convert to a percentage first	

Basics to memorise

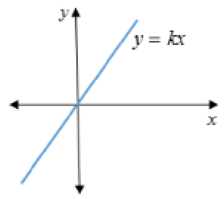
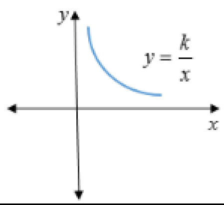
44.	Fraction	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
	Decimal	0.01	0.1	0.125	0.2	0.25	$0.\dot{3}$	0.5	$0.\dot{6}$	0.75
	Percentage	1%	10%	12.5%	20%	25%	$33.\dot{3}\%$	50%	$66.\dot{7}\%$	75%

Terminating and recurring decimals

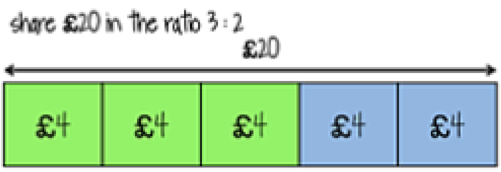
45.	Terminating decimal	Decimals that can be written exactly	e.g. 0.38
46.	Recurring decimal	Decimals where one digit or groups of digits are repeated	e.g. $0.\dot{7}$ = 0.7777… $0.\dot{8}5\dot{3}$ = 0.853853…
47.	Converting a recurring decimal to a fraction	1. Let x = recurring decimal. 2. Let n = the number of recurring digits. 3. Multiply the recurring decimal by 10^n. 4. Subtract (1) from (3) to eliminate the recurring part. 5. Solve for x, expressing your answer as a fraction in its simplest form.	
		$0.\dot{7}$ (one recurring digit) $x = 0.7777\ldots$ $10x = 7.777\ldots$ $10x - x = 7$ $9x = 7$ $x = \frac{7}{9}$	$1.2\dot{5}\dot{6}$ (two recurring digits) $x = 1.25656\ldots$ $100x = 125.6565\ldots$ $100x - x = 125.6565\ldots - 1.256565\ldots$ $99x = 124.4$ $x = \frac{124.4}{99} = \frac{1244}{990} = \frac{622}{495}$
48.	Converting a fraction to recurring decimals	Carry out the neccesary division using a calcualtor or bus stop division	e.g. $\frac{4}{7}$ means $4 \div 7$ $0.5\,7\,1\,4\,2\,8\,5\,7$ $7 \,\big)\, 4.{}^{4}0\,{}^{5}0\,{}^{1}0\,{}^{3}0\,{}^{2}0\,{}^{6}0\,{}^{4}0\,{}^{5}0$

Ratio and Proportion

49.	Ratio	A relationship between two or more quantities
50.	Unit ratio	Used to compare ratios, one of the parts is 1
		The only time it is permissible to have a decimal in a ratio
51.	Equivalent	Ratios that have the same simplified form are said to be equivalent

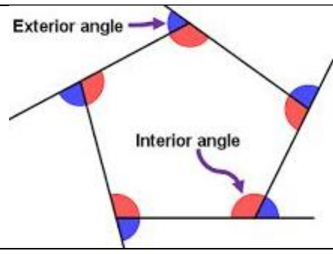
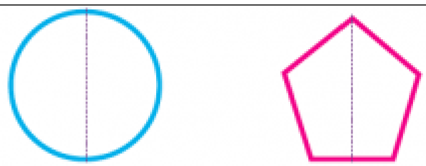
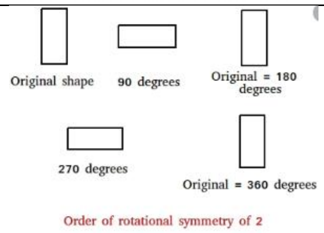
52.	Scale	A ratio that represents the relationship between a length on a drawing or a map and the actual length	
53.	Proportion	Compares a part with a whole	
54.	Direct proportion	Two quantities increase at the same rate Graph is a straight line that goes through the origin	$y \propto x$ $y = kx$ for a constant k 
55.	Inverse/indirect proportion	One variable increases at a constant rate as the second variable decreases	$y \propto \frac{1}{x}$ $y = \frac{k}{x}$ for a constant k 
56.	Proportional	A change in one is always accompanied by a change in the other	
57.	Constant of proportionality	Represented by k Its value stays the same	
58.	Share	Splitting into parts as defined by a ratio	
59.	Unitary method	Finding the value of 1 item then using this to find the value of any number of that item Use to work out which products give the best value for money	

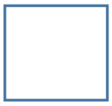
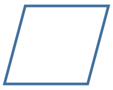
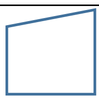
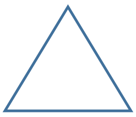
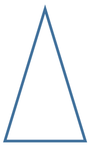
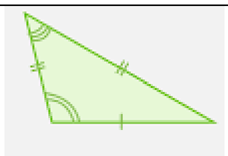
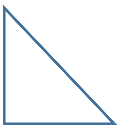
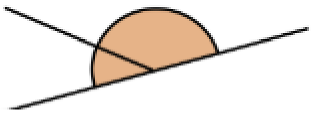
Working with ratios

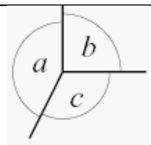
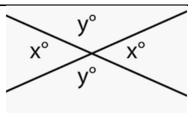
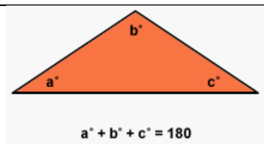
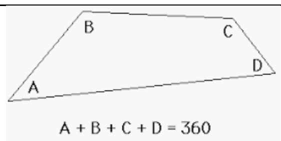
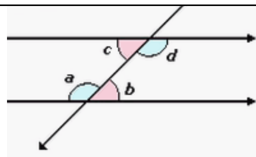
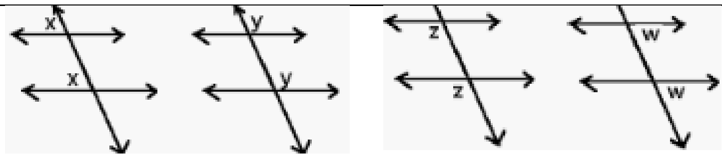
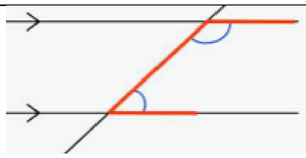
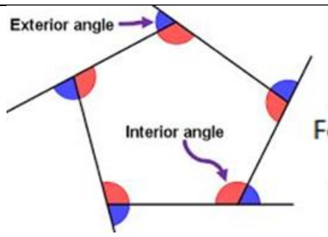
60.	Simplifying ratio	Divide all parts by the highest common factor All parts in the simplified version must be integers	e.g. 12:4 simplifies to 3:1 (divided by HCF of 4)
61.	Divide in a given ratio	Divide an amount so the ratio of the final values simplifies to the given ratio	

GCSE Mathematics
Higher
Unit 5

Shapes and angles - definitions

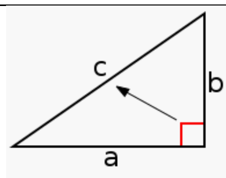
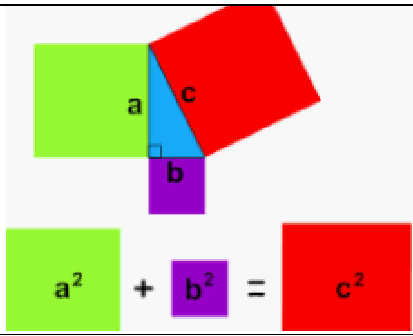
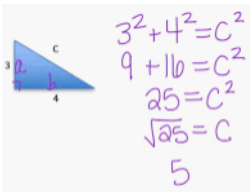
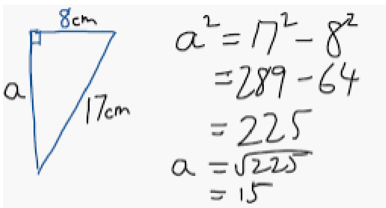
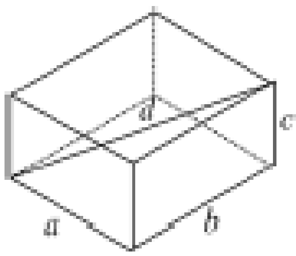
1.	Angle	A measure of turn, measured in degrees ◦	
2.	Protractor	Instrument used to measure the size of an angle	
3.	Acute angle	An angle less than 90◦	
4.	Right angle	A 90◦ angle	
5.	Obtuse angle	An angle more than 90◦ but less than 180◦	
6.	Reflex angle	An angle more than 180◦	
7.	Parallel lines	Lines that are equal distance apart that will never meet even when extended	
8.	Perpendicular lines	Lines that intersect at a right angle	
9.	Polygon	A 2D shape with straight lines only	
10.	Regular polygon	A polygon where: All sides are the same length All angles are the same size	
11.	Interior angles (I)	An angle inside a polygon	 For any polygon: $I + E = 180^0$
12.	Exterior angles (E)	An angle outside a polygon	
13.	Congruent	Shapes that are the same shapes and size, they are identical.	
14.	Similar	Shapes that are the same shape but are different sizes	
15.	Bisect	Cut in half	
16.	Tessellate	Fit together without leaving gaps	
17.	Symmetry	A shape has symmetry if a central line is drawn to show both sides are exactly the same. We call these lines of symmetry	
18.	Rotational symmetry	A shape has rotational symmetry when it looks the same after some rotation of less than a full turn	

Quadrilaterals (4 sided shapes)				
19.	Square		4 equal sides 4 equal angles 2 pairs of parallel sides Diagonals cross at right angles	4 lines symmetry Rotational symmetry order 4
20.	Rectangle		2 pairs of equal sides 4 right angles 3 pairs of parallel sides	2 lines of symmetry Rotational symmetry order 2
21.	Rhombus		4 equal sides 2 pairs of equal angles 2 pairs of parallel sides Diagonals cross at right angles	2 lines of symmetry Rotational symmetry order 2
22.	Parallelogram		2 pairs of equal sides 2 pairs of equal angles 2 pairs of parallel sides	0 lines of symmetry Rotational symmetry order 2
23.	Kite		2 pairs of equal sides 1 pair of equal angles 2 pairs of parallel sides Diagonals cross at right angles	1 line of symmetry Rotational symmetry order 1
24.	Trapezium		One pair of parallel lines	
25.	Isosceles trapezium		1 pair of parallel sides 1 pair of equal sides 2 pairs of equal angles	1 line of symmetry Rotational symmetry order 1
Triangles (3 sided shapes)				
26.	Equilateral		3 equal sides 3 equal angles	3 lines of symmetry Rotational symmetry order 3
27.	Isosceles		2 equal sides 2 equal angles	1 line of symmetry Rotational symmetry order 1
28.	Scalene		No equal sides No equal angles	
29.	Right-angled		1 right angle Can be scalene or isosceles	
Basic angle rules				
30.	Angles on a straight line add to 180°			

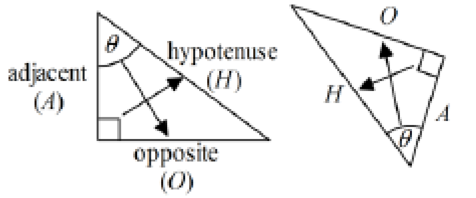
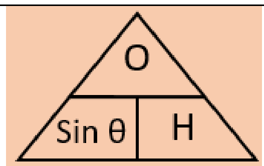
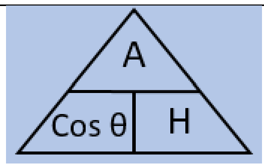
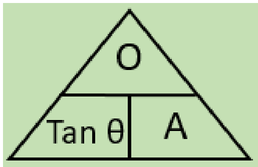
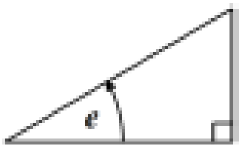
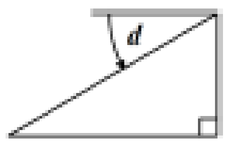
31.	Angles around a point add up to 360°	
32.	Vertically opposite angles are equal	
33.	Angles in a triangle add to 180°	
34.	Angles in a quadrilateral add up to 360°	
Angles on parallel lines		
35.	Alternate angles are equal	
36.	Corresponding angles are equal	
37.	Co-interior angles add up to 180°	
Angles in polygons		
38.	Interior and exterior angles add to give 180°	 For any polygon: $I + E = 180^0$
39.	Sum of interior angles	For a 'n' sided polygon Sum of interior angles = 180 x (n-2)

40.	Size of one interior angle	For a 'n' sided polygon Interior angle = $\frac{180\ x\ (n-2)}{n}$
41.	Sum of exterior angles	For all polygons, sum of exterior angles = 360°
42.	Regular polygons	Exterior angle = 360 ÷ number of sides Number of sides = 360 ÷ exterior angle Interior angle = 180 – exterior angle

Pythagoras' Theorem

43.	Hypotenuse	The longest side of a right-angled triangle It is always opposite the right angle	
44.	Right- angled triangle	A triangle that contains a right angle	
45.	Pythagoras' Theorem	$a^2 + b^2 = c^2$ Where c is the hypotenuse Where a and b are the two shorter sides	
46.	To find the hypotenuse (c)		• Square • Add • Square root
47.	To find a short side (a/b)		• Square • Subtract • Square root
48.	Pythagoras' in 3D	$a^2 + b^2 + c^2 = d^2$ $d^2 - b^2 - c^2 = a^2$	

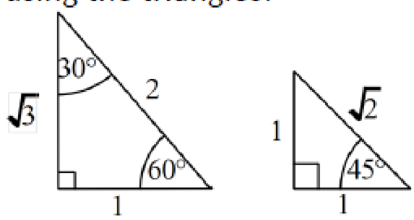
Trigonometry - Right angled – SOH CAH TOA

49.	Trigonometry	The ratios between the sides and angles of triangles		
50.	Labelling the triangle	*θ is the angle involved* H is the hypotenuse O is the opposite A is the adjacent		
51.	Sine	SOH		$Sin\ \theta = \frac{Opp}{Hyp}$ $\theta = Sin^{-1}\frac{Opp}{Hyp}$
52.	Cosine	CAH		$Cos\ \theta = \frac{Adj}{Hyp}$ $\theta = Cos^{-1}\frac{Adj}{Hyp}$
53.	Tangent	TOA		$Tan\ \theta = \frac{Opp}{Adj}$ $\theta = Tan^{-1}\frac{Opp}{Adj}$
54.	Exact Values	see below		
55.	Angle of elevation		Angle of depression	

54. Exact Values

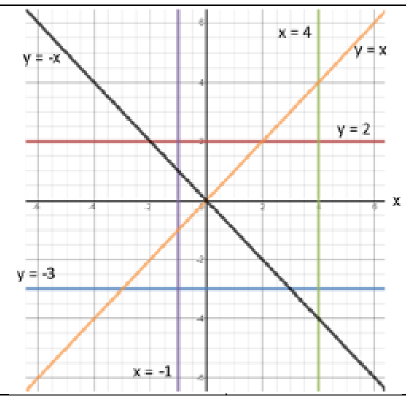
Θ	0°	30°	45°	60°	90°
Sin Θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos Θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan Θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

These can be found using the triangles:

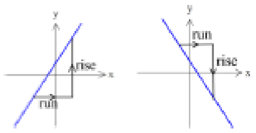
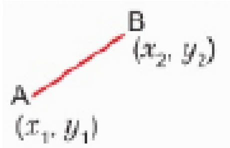


GCSE Mathematics Higher Unit 6

Graphs - definitions

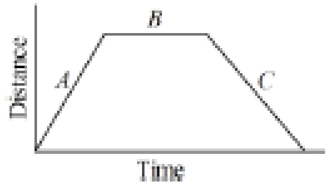
1.	Axis	A reference line on a graph	
2.	Axes	Plural of axis	
3.	Quadrant	A quarter of a graph separated by a axes	
4.	Coordinate	Used to show a position on a coordinate plane, (x, y)	
		First coordinate is the horizontal position, (x axis) and the second is the vertical position (y axis)	
5.	Origin	The point (0,0) on a set of axes	
6.	Plot	Mark a position or positions on a graph	
7.	y intercept	The y value where a graph crosses the y axis	where x=0
8.	x intercept	The x value where a graph crosses the x axis	where y=0
9.	Parallel	Lines that are equal distance apart that if extended will never meet	
10.	"y=" graph	Constant y coordinate	
		Will be parallel to the x axis	
11.	"x=" graph	Constant x coordinate	
		Will be parallel to the y axis	
12.	Linear function	An arithmetic sequence that can be represented by a straight line graph	
13.	Linear equation	An equation that produces a straight line graph	
14.	Equation of a line	$y = mx + c$ $m = gradient$ $c = y\ intercept$	$ax + by + c = 0$ Where a, b and c are integers

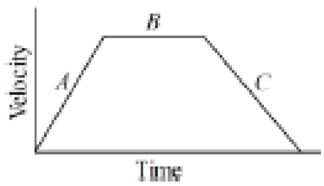
Coordinate geometry

15.	Gradient	The steepness of a graph $Gradient = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$	 This has a positive gradient This has a negative gradient
16.	Gradient between two points	If A = (x_1, y_1) and B = (x_2, y_2) The gradient of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$	
17.	Parallel lines	Have the same gradients	
18.	Perpendicular	Lines that are at right angles to one another Lines that are perpendicular are the negative reciprocal of one another If two lines are perpendicular, the product of their two gradients is -1	If a line has a gradient of m, the gradient of a line perpendicular to it will have a gradient of $-\frac{1}{m}$
19.	Mid-point	The coordinate half way between two point	If A = $(x_1,\ y_1)$ and B = $(x_2,\ y_2)$ the mid-point is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
20.	Distance between two points	Distance (d) between $(x_{1,}y_1)$ and $(x_{2,}y_2)$ can be found using the formula $d = \sqrt{(x_2 - x_{1,})^2 + (y_2 - y_{1,})^2}$	

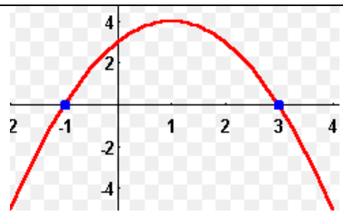
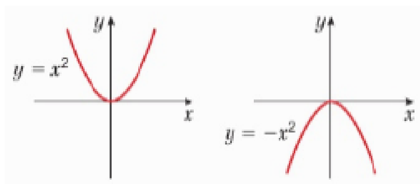
Real life graphs

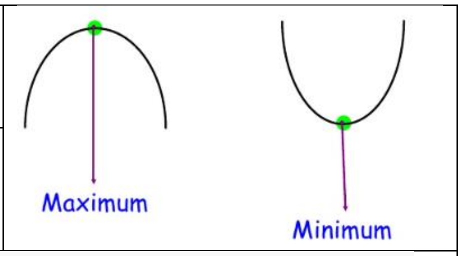
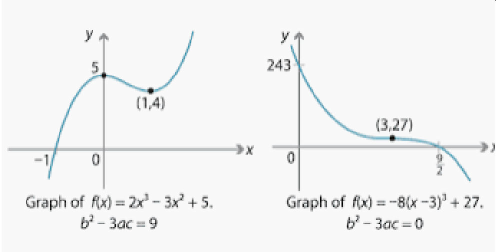
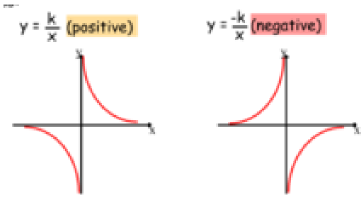
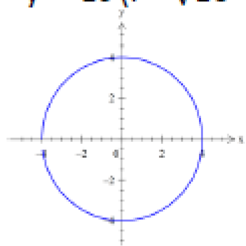
21.	Steady speed	Travelling the same distance each minute
22.	Velocity	Speed in a particular direction
23.	Rate of change	Shows how a variable changes over time
24.	Acceleration	How fast velocity changes; measured in m/s^2 or km/s^2 etc

<table>
<tr><th colspan="3">Distance - Time graphs</th></tr>
<tr><td>25.</td><td>Represent a journey</td><td rowspan="7">

A = steady speed,
B = no movement,
C = steady speed back to start</td></tr>
<tr><td>26.</td><td>Vertical axis represents the distance from the starting point</td></tr>
<tr><td>27.</td><td>Horizontal axis represents the time taken</td></tr>
<tr><td>28.</td><td>Straight lines mean constant speed</td></tr>
<tr><td>29.</td><td>Horizontal lines mean no movement</td></tr>
<tr><td>30..</td><td>Gradient = speed</td></tr>
<tr><td>31.</td><td>Average speed = =$\frac{total\ distance}{total\ time}$</td></tr>
<tr><th colspan="3">Velocity – Time graphs</th></tr>
<tr><td>32.</td><td>Represents the speed at given times</td><td rowspan="6">

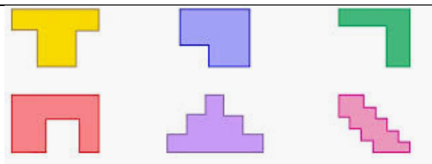
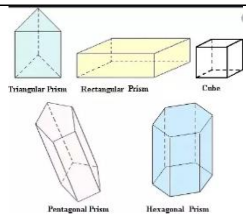
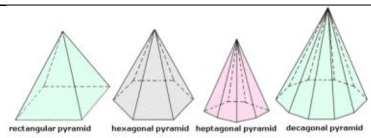
A = steady acceleration,
B = constant speed,
C = steady deceleration back to a stop</td></tr>
<tr><td>33.</td><td>Straight lines mean constant acceleration or deceleration</td></tr>
<tr><td>34.</td><td>Horizontal change means no change in velocity e.g. constant speed</td></tr>
<tr><td>35.</td><td>Positive gradient-= acceleration</td></tr>
<tr><td>36.</td><td>Negative gradient = deceleration</td></tr>
<tr><td>37.</td><td>Distance travelled = area under the graph</td></tr>
</table>

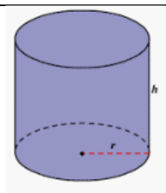
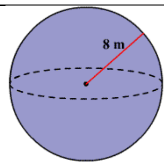
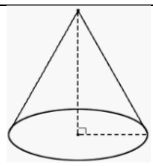
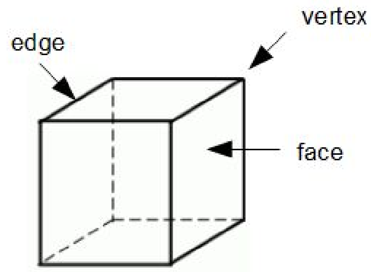
<table>
<tr><th colspan="4">Quadratic, cubic and other graphs</th></tr>
<tr><td>38.</td><td>Quadratic expression</td><td>An expression where the highest index is 2</td><td>e.g. $2x^2 + 2x + 2$</td></tr>
<tr><td rowspan="3">39.</td><td rowspan="3">Roots</td><td>Solutions to a quadratic equation/function
$ax^2 + bx + c = 0$</td><td rowspan="3">
</td></tr>
<tr><td>The x values where the graph crosses the x axis</td></tr>
<tr><td>A quadratic can have 0, 1 or 2 roots</td></tr>
<tr><td rowspan="3">40.</td><td rowspan="3">Quadratic graph</td><td>Curved shaped called a parabola</td><td rowspan="3">
</td></tr>
<tr><td>A positive x^2 will give a ‘∪’ shape</td></tr>
<tr><td>A negative x^2 will give a ‘∩’ shape</td></tr>
</table>

41.	Turning points	The point where a curve turns in the opposite direction Can be called a minimum or maximum	
42.	Cubic	General form of $ax^3 + bx^2 + cx + d = 0$ Can have 1, 2 or 3 roots	
43.	Asymptote	A line a graph will get very close to but will not touch	
44.	Reciprocal	General form of $y = \frac{k}{x}$ where k is a number Has two asymptotes	
45.	Circle	With centre $(0,0)$ and radius, r $x^2 + y^2 = r^2$	. $x^2 + y^2 = 16$ $(r = \sqrt{16} = 4)$ 

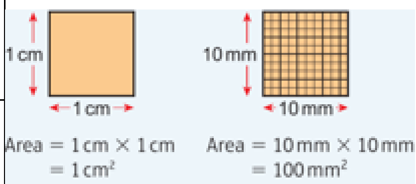
GCSE Mathematics
Higher
Unit 7

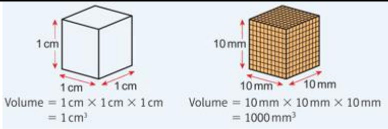
2D and 3D shapes: definitions

1.	Dimension	The size of something in a particular direction e.g. height, depth, length, width
2.	2D shape	A shape that has length/height and a width but no depth
3.	3D shape	A shape that depth as well as length/height and width
4.	Polygon	A 2D shape with straight lines only
5.	Regular polygon	A polygon where: All sides are the same length All angles are the same size
6.	Compound shape	A shape made up of two or more simple shapes
7.	Rectilinear shape	A shape where all of its sides meet at right angles 
8.	Perimeter	The distance around the outside of a 2D shape
9.	Area	The space inside a 2D shape
10.	Surface area	The total area of all the faces of a 3D shape
11.	Volume	The space inside a 3D shape
12.	Capacity	The amount of fluid a 3D object can hold
13.	S.I. Units	Standard units of measurement used by scientists across the world
14.	Metric units	Standard units of measurement that vary by powers of 10
15.	Imperial units	Older units of measurement, some of which are still common e.g. miles, gallons
16.	Cross section	The shape we get when cutting straight through a 3D shape
17.	Prism	A 3D shape that has a constant cross section through its length 
18.	Pyramid	A 3D shape with a polygon as its base and triangular sides that meet at the top 

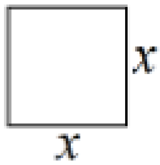
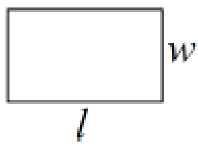
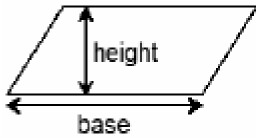
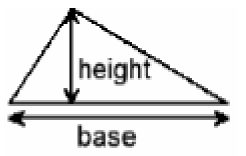
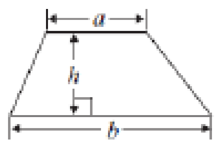
19.	Cylinder	A prism with two circular ends connected by a curved surface	
20.	Sphere	A 3D shape where all points on the surface are the same distance from the centre	
21.	Spherical	Means in the shape of a sphere	
22.	Cone	A 2D shape that has a circular base joined to a point by a curved side	
23.	Face	A flat surface of a 3D shape (can be curved)	
24.	Edge	A line segment where two faces meet	
25.	Vertex	A point where two or more edges meet	
26.	Vertices	Plural of vertex	

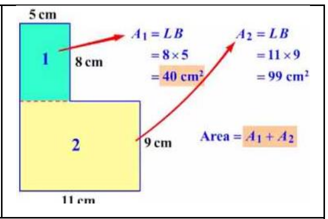
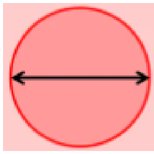
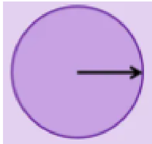
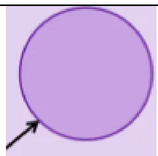
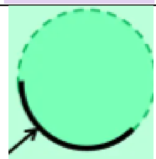
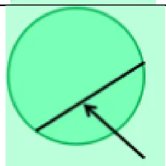
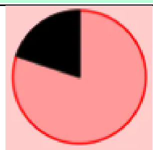
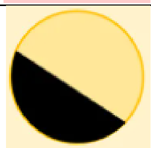
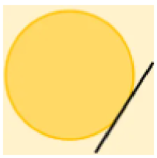
Measures

27.	Units of time	Standard units of time are seconds, minutes, hours, days, years			
		60 seconds = 1 minute	60 minutes = 1 hour	24 hours = 1 day	365 days = 1 year
28.	Units of mass	Metric units of mass are milligrams, grams, kilograms and tonnes			
		1000mg = 1g	1000g = 1kg	1000kg = 1 tonne	
29.	Units of length	Metric units of length are millimetres, centimetres, metres and kilometres			
		10mm = 1cm	100cm = 1m	1000m = 1km	
30.	Units of area	Metric units of length are millimetres^2, centimetres^2, metres^2 and kilometres^2			
		$1cm^2 = 100mm^2$			
		$1m^2 = 1000cm^2$			

31.	Units of volume	Metric units of length are millimetres^3, centimetres^3, metres^3 and kilometres^3		
		1cm^3 = 1000mm^3		
		1m^3 = 1000000cm^3		
32.	Units of capacity	Metric units of capacity are millilitres, centilitres and litres		
		10*ml* = 1*cl*	1000*ml* = 100*cl* = 1*l*	
33.	Capacity and volume conversions	1cm^3 = 1*ml*	1000cm^3 = 1*l*	

2D Shapes

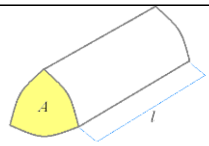
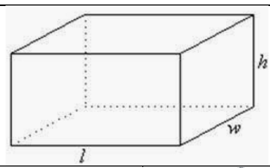
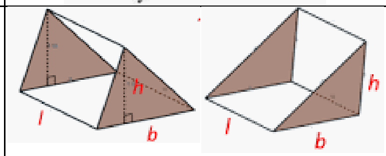
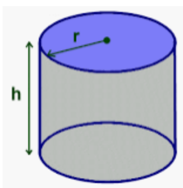
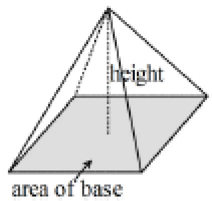
34.	Square	Area = $l \times w$ or l^2 as length and width are equal	
35.		Perimeter = $l + l + l + l$ or $4l$	
36.	Rectangle	Area = $l \times w$	
37.		Perimeter = $l + l + w + w$ or $2l + 2w$	
38.	Parallelogram	Area = $b \times h$	
39.	Triangle	Area = $\frac{b \times h}{2}$ or $\frac{1}{2} \times b \times h$	
40.	Trapezium	Area = $\frac{a+b}{2} \times h$ or $\frac{1}{2}(a+b) \times h$	

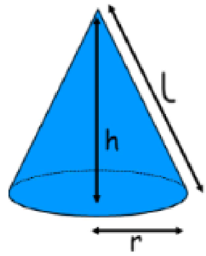
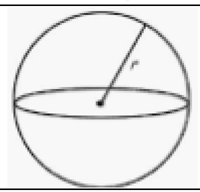
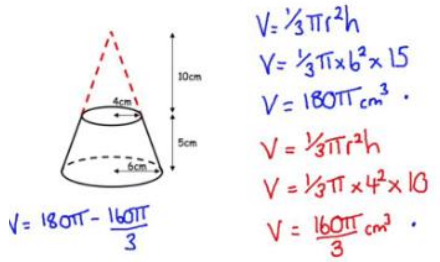
41.	Compound shape	To find the area, split up into simple shapes, find each area and add together. To find the perimeter, find any missing sides than add all the sides together.	
Circles			
42.	Diameter	A straight line from edge to edge passing through the centre Double the size of the radius	
43.	Radius	A straight line from the centre to the edge Half the size of the diameter	
44.	Radii	The plural of radius	
45.	Circumference	Distance around the outside of the circle	
46.	Arc	Part of the circumference	
47.	Chord	A line within a circle where each end touches the edge	
48.	Sector	The region created by two radii and an arc	
49.	Segment	The region created by a chord and an arc	
50.	Tangent	A line outside the circle which only touches the circumference at one point	
51.	Semi -circle	Half a full circle	

Area and circumference of circles formulae

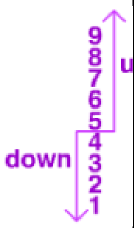
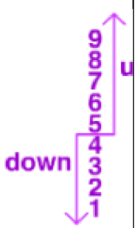
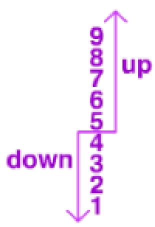
52.	Pi (π)	Constant ratio linking the circumference and diameter of a circle 3.14159265…	
53.	Circumference of a circle	$C = \pi d$	Alternatively, using relationship between r and d $C = 2\pi r$
54.	Arc length	$\frac{x}{360} \times \pi d$	Where x is the angle at the centre
55.	Perimeter of a sector	$\left(\frac{x}{360} \times \pi d\right) + 2r$	This represents the arc length plus the two radii
56.	Area of a circle	$A = \pi r^2$	
57.	Area of a sector	$\frac{x}{360} \times \pi r^2$	

3D shapes: volume

58.	Prism	Volume = $area\ of\ cross\ section \times length$	
59.	Cuboid	Volume = $area\ of\ cross\ section \times length$ Volume = $length \times width \times height$	
60.	Triangular prism	Volume = $area\ of\ cross\ section \times length$ Volume = $\frac{1}{2} \times base \times height \times length$	
61.	Volume of a cylinder	$V = \pi r^2 h$	
62.	Surface area of a cylinder	$Total\ surface\ area = 2\pi r^2 + \pi dh$	
63.	Volume of a pyramid	$V = \frac{1}{3} \times area\ of\ base \times perpendicular\ height$	

64.	Volume of a cone	$V = \frac{1}{3} \times \pi r^2 h$	
65.	Surface area of a cone	$Curved\ surface\ area = \pi r l$ $Total\ surface\ area = \pi r^2 + \pi r l$	
66.	Volume of a sphere	$V = \frac{4}{3} \times \pi r^3$	
67.	Surface area of a sphere	$Total\ surface\ area = 4\pi r^2$	
68.	Volume of a frustum	Find the volume of the whole cones and subtract the volume of the smaller cone to get the volume of the frustum	

Accuracy and Bounds

69.	Integer	A whole number and the negative equivalents.		
70.	Rounding	Changing a number to a simpler, easy to use value		
71.	Round to a given number of decimal places	• Count the number of decimal places you need. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. 36.3486343 $36.3\|486343$ To 1 d.p. is 36.3 $36.34\|86343$ To 2 d.p. is 36.35 $36.348\|6343$ To 3 d.p. is 36.349
72	Round a large number to a given number of significant figures	• Count the number of digits you need from the left. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down. • Replace remaining digits with zeros as place holders.		e.g. $324\,627\,938$ $3\|24627938$ To 1 s.f. is 300000000 $32\|4627938$ To 2 s.f. is 320000000 $324\|627938$ To 3 s.f. is 325000000
73.	Round a small number to a given number of significant figures	• Zeros are not significant until after the first non-zero number. • Find the first non-zero and count the number of digits you need from there. • Look at the number to the right of that digit to decide if it should round up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. 0.0034792 $0.003\|4792$ To 1 s.f. is 0.003 $0.0034\|792$ To 2 s.f. is 0.0035 $0.00347\|92$ To 3 s.f. is 0.00348

74.	Estimating	• Round each number to 1 significant figure before doing any calculations. • It is acceptable to round one or more numbers in the calculation to a greater accuracy than 1 sig. fig. if this makes the calculation easier. • DO NOT round the answer!	e.g. Estimate: $\frac{\mathbf{3.91 \times 8789.8}}{\mathbf{620.9 \times 0.492}}$ $\frac{3.91 \times 8789.8}{620.9 \times 0.492} \approx \frac{4 \times 9000}{600 \times 0.5}$ $\approx \frac{3600}{300}$ $\approx \mathbf{120}$
75.	Truncation	Approximating a number by ignoring all decimal points after a certain point without rounding	e.g. 5.6 would be 5 when truncated
76.	Error interval	Measurements measured to the nearest unit may be up to half a unit smaller or larger than the rounded value	e.g. If 5.6 is rounded correct to the nearest 1dp then the interval is $5.55 \leq x < 5.65$
77.	Upper bound	The upper bound is half a unit greater than the rounded number	e.g. the upper bound of 5.6 when measured to the nearest 1dp is 5.65
78.	Lower Bound	The lower bound is half a unit less than the rounded number	e.g. the lower bound of 5.6 when measured to the nearest 1dp is 5.55
79.	Appropriate accuracy	The accuracy when both the upper and lower bound are rounded by the same amount and give the same value e.g. If UB = 12.3512 and LB = 12.3475 To 1dp: UB = 12.4 and LB- 12.3 To 2dp: UB = 12.35 and LB – 12.35 To 3dp: UB = 12.351 and LB =12.348	Here the appropriate accuracy is 2 dp