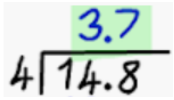
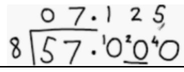
GCSE Mathematics Higher Unit 1

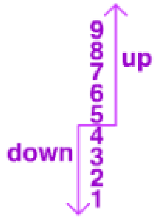
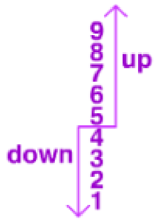
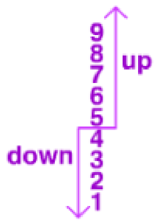
Definitions

Integer	A whole numbers and the negative equivalents.	
Positive	Greater than zero.	
Negative	Less than zero.	
Decimal	A number with digits after the decimal point.	
Operations	Symbols describing how to combine numbers. $\times \rightarrow$ Multiply, $\div \rightarrow$ Divide, $+ \rightarrow$ Add, $- \rightarrow$ Subtract,	
Multiplications terms	*Multiplicand:* The number being multiplied. *Multiplier:* The number that we are multiplying by. *Product:* The result of the multiplication operation.	multiplicand $2 \times 3 = 6$ multiplier product
Division terms	*Dividend:* The number being divided. *Divisor:* The number we are dividing by. *Quotient:* The result of the division operation.	Dividend $40 \div 8 = 5$ Divisor Quotient 6 ← quotient $4\overline{)24}$ ← dividend divisor
Inverse operations	The operation used to reverse the original operation.	$+$ and $-$ are inverses $\times$ and $\div$ are inverses Square and square root are inverses Cube and cube root are inverses
Order of Operations	The order in which operations should be done.	B – Brackets I – Indices DM – Division & Multiplication AS – Addition & Subtraction
$\neq$	Not equal to.	
Inclusive	Includes the first and last numbers given.	
Index Form	A number written as a base to the power of something.	Base 2^7 Exponent Power Index
Prefix	The first part of a word, sometimes separated from the rest of the word by a hyphen.	
Standard Form	A number written in the form: $A \times 10^n$, where A is between 1 and 10.	
Scientific Notation	Another name for Standard Form.	
Surd	An method of writing non square or cube numbers as exact numbers in root form .	e.g. $\sqrt{4}$ is NOT a surd because $\sqrt{4} = 2$ $\sqrt{7}$ IS a surd because it is between 2 and 3
Fraction	Represents a proportion or part of a whole.	e.g. $\frac{4}{5}$
Numerator	The number or term on top of the fraction.	$\frac{Numerator}{Denominator}$
Denominator	The number or term on the bottom of the fraction.	
Rationalise the denominator	Eliminate a surd denominator in a fraction.	

1a. Calculations, checking and rounding (N2, N3, N5, N14, N15)

i)	Add & subtract decimals	Use the column method making sure making sure the decimal points are vertically aligned	$3.8 - 1.26 \rightarrow$ $\begin{array}{r} 3.\overset{7}{\cancel{8}}{}^{1}0 \\ -\ 1.26 \\ \hline 2.54 \end{array}$

ii)	Multiply decimals	Multiply the integers and correct place value	Calculate: $\mathbf{4.32 \times 20.8}$ Use: $432 \times 208 = 89856$ So: $4.32 \times 20.8 = 89.856$ *2 dp* *1 dp* *3dp*
iii)	Divide decimals	*<u>Dividing a decimal by an integer</u>*: Use short division ensuring that a decimal point is placed vertically above the decimal point in the dividend.	
		<u>Division with a decimal remainder</u>: add a decimal point and additional zero's after the dividend to allow you to continue the short division as above.	Calculate: $\mathbf{57 \div 8}$ Use: 
		<u>Dividing by a decimal:</u> Multiply dividend and divisor by 10, 100 , 1000 so that the divisor becomes an integer then complete short division as above. <u>N.B. Do not place value after the calculation!</u>	Calculate: $\mathbf{6.488 \div 0.8}$ $\times 10 \quad \times 10$ Use: $64.88 \div 8 = 8.11$ So: $6.488 \div 0.8 = \mathbf{8.11}$
iv)	Multiply any number between 0 and 1	Use the methods described above in: ii) Multiply decimals *N.B.* Value of the product will be smaller than the value of the multiplicand if the multiplier is between 0 and 1 and vice-versa.	$\mathbf{12} \times 0.2 = \mathbf{6}$ And: $0.2 \times \mathbf{12} = \mathbf{6}$
	Divide any number between 0 and 1	Use the methods described above in: iii) Divide decimals *N.B.* Value of the quotient will be greater than the value of the dividend if the divisor is between 0 and 1.	$\mathbf{12} \div 0.2 = \mathbf{60}$
v)	Use one calculation to find the answer to another	Given: $a \times b = c$ Then: $c \div b = a$ and $c \div a = b$ Adjust place value if necessary.	If: $\mathbf{19 \times 24 = 456}$ $456 \div 24 = 19$ $456 \div 19 = 24$ $1.9 \times 24 = 45.6$ $456 \div 190 = 2.4$ $19 \times 240 = 4560$
vi)	Use the product rule for counting: multiple groups	There are $\boldsymbol{n}$ different options available from group A and $\boldsymbol{m}$ different options available from group B. The number of possible combinations that can occur when choosing one option from Group A <u>and</u> one option from Group B is given by: $\boldsymbol{n \times m}$	e.g. A restaurant serves 4 different starter and 5 different main courses. How many combinations of start and main course could you choose? $\mathbf{4 \times 5 = 30}$
	Use the product rule for counting: one group with repeats	There are $\boldsymbol{n}$ possible options available from a single group and the same option can be selected multiple times. The number of possible combinations that can occur when choosing $\boldsymbol{m}$ options is given by: n^m	e.g. A combination lock has 3 wheels with the numbers 1 to 8 on each wheel. How many different combinations are possible? $\mathbf{8^3 = 512}$
	Use the product rule for counting: one group without repeats	There are n possible options from a single group and each options can be selected once only. The number of possible outcomes that can occur when choosing m options is given by: $n \times (n-1) \times (n-2) \times \ldots\ldots\ldots \times (n-m+1)$	e.g. 12 people run a marathon, how many combinations of gold, silver and bronze medal winners are there? $\mathbf{12 \times 11 \times 10 = 1320}$

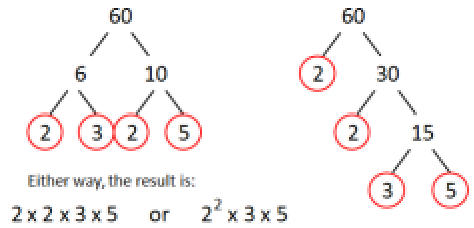
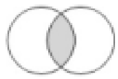
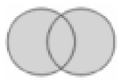
vii)	Round to a given number of decimal places	• Count the number of decimal places you need. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. $\mathbf{36.3486343}$ 36.3|486343 To 1 d.p. is $\mathbf{36.3}$ 36.34|86343 To 2 d.p. is $\mathbf{36.35}$ 36.348|6343 To 3 d.p. is $\mathbf{36.349}$
ii)	Round a large number to a given number of significant figures	• Count the number of digits you need from the left. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down. • Replace remaining digits with zeros as place holders.		e.g. $\mathbf{324\ 627\ 938}$ 3|24627938 To 1 s.f. is $\mathbf{300000000}$ 32|4627938 To 2 s.f. is $\mathbf{320000000}$ 324|627938 To 3 s.f. is $\mathbf{325000000}$
ix)	Round a small number to a given number of significant figures	• Zeros are not significant until after the first non-zero number. • Find the first non-zero and count the number of digits you need from there. • Look at the number to the right of that digit to decide if it should round up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. $\mathbf{0.0034792}$ 0.003|4792 To 1 s.f. is $\mathbf{0.003}$ 0.0034|792 To 2 s.f. is $\mathbf{0.0035}$ 0.00347|92 To 3 s.f. is $\mathbf{0.00348}$
x)	Estimating	• Round each number to 1 significant figure before doing any calculations. • It is acceptable to round one or more numbers in the calculation to a greater accuracy than 1 sig. fig. if this makes the calculation easier. • DO NOT round the answer!		e.g. Estimate: $\frac{\mathbf{3.91 \times 8789.8}}{\mathbf{620.9 \times 0.492}}$ $\frac{3.91 \times 8789.8}{620.9 \times 0.492} \approx \frac{4 \times 9000}{600 \times 0.5}$ $\approx \frac{3600}{300}$ $\approx \mathbf{120}$

1b. Indices, roots, reciprocals and hierarchy of operations (N2, N3, N6, N7, N14)

X i)	Use index notation for positive powers of 10	• Count how many zero's there are after the 1 and write 10 to the power of this number. • Write a 1 followed by the same number of zero's as the power 10 is raised to.	e.g. $\mathbf{10\ 000\ 000 = 10^7}$ e.g. $\mathbf{10^2 = 100}$
ii)	Use index notation for negative powers of 10	• Count how many zero's there are in front of the 1 and write 10 to the power of the negative of this number. • Use the positive of the power 10 is raised to and write a 1 with this number of zero's in front with a decimal point after the first.	e.g. $\mathbf{0.000\ 000\ 1 = 10^{-7}}$ e.g. $\mathbf{10^{-2} = 0.01}$

<table>
<tr><td rowspan="5">iii)</td><td>Recognise common powers</td><td colspan="3">Recall that the positive power of a number tells us how many times to use that number in a multiplication.</td><td>e.g. $3^4 = 3 \times 3 \times 3 \times 3$
e.g. $7^2 = 7 \times 7$</td></tr>
<tr><td>Powers of 2</td><td colspan="4">$2^1 = \mathbf{2}, 2^2 = \mathbf{4}, 2^3 = \mathbf{8}, 2^4 = \mathbf{16}, 2^5 = \mathbf{32}, 2^6 = \mathbf{64}, 2^7 = \mathbf{128}, 2^8 = \mathbf{256}. 2^9 = \mathbf{512}, 2^{10} = \mathbf{1024}$</td></tr>
<tr><td>Powers of 3</td><td colspan="4">$3^1 = \mathbf{3}, 3^2 = \mathbf{9}, 3^3 = \mathbf{27}, 3^4 = \mathbf{81}, 3^5 = \mathbf{243}$</td></tr>
<tr><td>Powers of 4</td><td colspan="4">$4^1 = \mathbf{4}, 4^2 = \mathbf{16}, 4^3 = \mathbf{64}, 4^4 = \mathbf{256}, 4^5 = \mathbf{1024}$</td></tr>
<tr><td>Powers of 5</td><td colspan="4">$5^1 = \mathbf{5}, 5^2 = \mathbf{25}, 5^3 = \mathbf{125}, 5^4 = \mathbf{625}$</td></tr>
<tr><td>iv)</td><td>Estimate roots of any given positive number</td><td colspan="3">• Identify the square (or cube) numbers immediately above and below the number we are trying to find the square (or cube) root of.
• The desired root must lie between the integer roots of the square numbers immediately above and below.</td><td>e.g. Between which two integers does $\sqrt{42}$ lie?
• Next square number is $\mathbf{49}$.
• Previous square number is $\mathbf{36}$.
• $\sqrt{36} = \mathbf{6}, \sqrt{49} = \mathbf{7}$
• So: $\sqrt{42}$ lies between : $\mathbf{6}$ & $\mathbf{7}$</td></tr>
<tr><td rowspan="3">v)</td><td>Find the value of calculations involving positive indices</td><td colspan="3">Recall that a positive power of a number tells us how many times to use that number in a multiplication.</td><td>e.g. $3^4 = 3 \times 3 \times 3 \times 3$
e.g. $7^2 = 7 \times 7$</td></tr>
<tr><td>Find the value of calculations involving negative indices</td><td colspan="2">To calculate a negative power:
• Calculate the equivalent positive power.
• Then take the reciprocal.</td><td>$a^{-n} = \frac{1}{a^n}$</td><td>e.g. Calculate $\mathbf{4^{-3}}$.
• $4^3 = 64$
• $4^{-3} = \frac{\mathbf{1}}{\mathbf{64}}$</td></tr>
<tr><td>Find the value of calculations involving fractional indices</td><td colspan="2">The denominator of the fractional power gives the type of root to evaluate.</td><td>$a^{\frac{1}{n}} = \sqrt[n]{a}$</td><td>e.g. $\mathbf{64^{\frac{1}{2}}} = \sqrt{\mathbf{64}} = \mathbf{8}$
e.g. $\mathbf{125^{\frac{1}{3}}} = \sqrt[3]{\mathbf{125}} = \mathbf{5}$</td></tr>
<tr><td rowspan="2">vi)</td><td rowspan="2">Use powers of 0 and 1</td><td colspan="2">Anything to the power of $0 = 1$</td><td>$a^0 = 1$</td><td>e.g. $\mathbf{5^0} = \mathbf{1}$</td></tr>
<tr><td colspan="2">Anything to the power $1 =$ itself</td><td>$a^1 = a$</td><td>e.g. $\mathbf{5^1} = \mathbf{5}$</td></tr>
<tr><td rowspan="3">vii)</td><td rowspan="3">Use index laws to simplify or evaluate numerical expressions</td><td>Multiplication</td><td>• Add the powers</td><td>$a^m \times a^n = a^{m+n}$</td><td>e.g. $\mathbf{2^2} \times \mathbf{2^3} = \mathbf{2^5}\ (= \mathbf{32})$</td></tr>
<tr><td>Division</td><td>• Subtract the powers</td><td>$a^m \div a^n = a^{m-n}$</td><td>e.g. $\mathbf{3^9} \div \mathbf{3^4} = \mathbf{3^5}\ (= \mathbf{243})$</td></tr>
<tr><td>Brackets</td><td>• Multiply the powers</td><td>$(a^m)^n = a^{mn}$</td><td>e.g. $(\mathbf{7^4})^{\mathbf{3}} = \mathbf{7^{12}}$</td></tr>
</table>

1c. Factors, multiples and primes (N3, N4)

i)	Factors	A factor is a number that divides into another number	e.g. factors of 6: 1, 2, 3 and 6
ii)	Multiples	A multiple is a number from the times tables	e.g. multiples of 4: 4, 8, 12, 16, 20, ………
iii)	Prime number	A prime number is a number with exactly 2 factors	
		2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97	
iv)	Product	The answer when two or more numbers are multiplied together.	e.g. Product of 3 & 7: $3 \times 7 = \mathbf{21}$
v)	Prime factor decomposition	Writing a number as a *product of its prime factors*	
vi)	Highest common factor (HCF)	The highest number that divides exactly into two or more numbers.	 e.g. The HCF of 12 & 8: $\mathbf{4}$
vii)	Lowest common multiple (LCM)	The smallest positive number that is a multiple of two or more numbers.	 e.g. The LCM of 12 & 8: $\mathbf{24}$

1d. Standard form (N9)

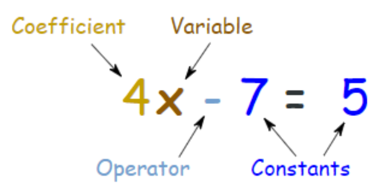
i)	Convert a small number to standard form	• Count the number of zero's in front of the first significant figure (including the one in front of the decimal point). • The power of ten is negative followed by this number.	e.g. $\mathbf{0.00000037}$ $= \mathbf{3.7 \times 10^{-7}}$
ii)	Convert a large number into standard form	• Count the number of place value position there are after the first significant figure. • The power of ten is positive followed by this number.	e.g. $\mathbf{147\ 100\ 000\ 000}$ $= \mathbf{1.47 \times 10^{11}}$
iii)	Converting to a small ordinary number	• Look at the digit after the negative in the power of 10. • Write this may zero's in front of the first sig. fig. • Reposition the decimal place between the first and second zero.	e.g. $\mathbf{2.4 \times 10^{-6}}$ $= \mathbf{0.0000024}$
iv)	Adding or subtracting numbers in standard form	• Convert the numbers to ordinary numbers. • Add. • Convert the sum to standard form.	e.g. $(\mathbf{2.3} \times \mathbf{10^4}) + (\mathbf{6.4} \times \mathbf{10^3})$ $= 23000 + 6400$ $= 29400$ $= \mathbf{2.94} \times \mathbf{10^4}$

v)	Multiplying numbers in standard form	• Multiply the numbers between one and 10 at the front. • Use index law for multiplication for the powers of 10. • If necessary increase the power of ten by one to ensure the initial number is between 1 and 10.	e.g. $(\mathbf{4.5} \times \mathbf{10^3}) \times (\mathbf{3} \times \mathbf{10^5})$ $= 13.5 \times 10^{3+5}$ $= 13.5 \times 10^8$ $= \mathbf{1.35} \times \mathbf{10^9}$
vi)	Dividing numbers in standard form	• Divide the numbers between one and 10 at the front. • Use index law for division for the powers of 10. • If necessary decrease the power of ten by one to ensure the initial number is between 1 and 10.	e.g. $(\mathbf{2.5} \times \mathbf{10^{11}}) \div (\mathbf{5} \times \mathbf{10^{13}})$ $= 0.5 \times 10^{-2}$ $= \mathbf{5} \times \mathbf{10^{-3}}$
1d. Surds (N8)			
i)	Multiply	$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\sqrt{a} \times \sqrt{a} = a$	e.g. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ and $\sqrt{3} \times \sqrt{3} = 3$
ii)	Divide	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	e.g. $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$
iii)	Add and subtract	$\sqrt{a} + \sqrt{b}$ cannot simplify	e.g. $\sqrt{3} + \sqrt{2} = \sqrt{3} + \sqrt{2}$
		But $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$	e.g. $5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$
iv)	Simplify	$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$	e.g. $\sqrt{50} + \sqrt{18} = 5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$
v)	Rationalise the denominator	Multiply numerator and denominator (use equivalent fractions) by whatever will result in the denominator simplifying to an integer.	e.g. $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{7}{\sqrt{7}}$ e.g. $\frac{1}{5+\sqrt{2}} = \frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{5-\sqrt{2}}{3}$

GCSE Mathematics Higher Unit 2

Algebra: the basics

Definitions

1.	Variable	A letter representing a varying or unknown quantity.	
2.	Coefficient	A number which multiplies a variable e.g. 4 is the coefficient in 4a	
3.	Term	One part of an expression/equation/formula Can involve multiplying and dividing coefficients and variables Separated from other terms by addition and subtraction	e.g. $4c$ $\frac{w}{5}$
4.	Like terms	Terms that have the same variable but have different coefficients	e.g. $c + 4c$ are like terms c^2 and c^3 are not like terms
5.	Constant	A fixed value. A number on its own or sometimes a letter such as a, b or c to represent a fixed number.	
6.	Expression	One or a group of terms. Can include variables, constants, operators and grouping symbols. No 'equals' sign	e.g. $3y - 3$ $3y^2 + y^3$
7.	Equation	Contains an 'equals' sign, = Has at least one variable	e.g. $3y - 3 = 12$
8.	Formula	A special type of equation that shows the relationship between a set of variables	
9.	Formulae	Plural of 'formula'	
10.	Identity	An equation that is true no matter what values are chosen, $\equiv$	e.g. $3y \equiv 2y - y$ for any value of y.
11.	Subject	The variable on its own on one side of the equals sign.	
12.	Substitute	Replace a variable with a number.	$a = 3, b = 2$ and $c = 5$. **Find:** 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$
13.	Simplify	Minimising the size of an expression	

14.	Factorise	Splitting an expression into a product of factors	
15.	Expand	Removing brackets by using multiplication	
16.	Solve	Find the value of an unknown	
Algebraic Notation			
17.	Adding like terms	Add the coefficients	$b + 2b = 3b$
18.	Subtracting like terms	Subtract the coefficients	$5b - 4b = b$
19.	Multiplying like terms	If the base is the same, add the powers	$b \times b = b^2$
20.	Dividing terms	If the base is the same, subtract the powers	$b^5 \div b^2 = b^3$
21.	Adding different terms	Cannot combine if the terms are different.	$b + 2c = b + 2c$
22.	Subtracting different terms	Cannot combine if the terms are different.	$3c - 4 = 3c - 4$
23.	Multiplying different terms	Combine with no '×' sign	$d \times e = de$
24.	Multiplying different terms with coefficients	Combine with no '×' sign, multiply the coefficients	$2d \times 3e = d6e$
25.	Dividing different terms	Write as fractions with no '÷' sign	$3d \div e = \frac{3d}{e}$
26.	Dividing different terms with coefficients	Write as fractions with no '÷' sign, simplify the coefficients where possible.	$14d \div 7e = \frac{2d}{e}$
Expanding (single brackets)			
27.	Multiply all the terms inside the bracket, by the term on the outside.		
28.	$3(a + 4) = 3a + 12$	$\begin{array}{c\|c\|c} \times & 2x & -3 \\ \hline 2x & 4x^2 & -6x \end{array}$	$4x^2 - 6x$
Factorising (single brackets)			
29.	• Find the highest common factor of the terms • This goes outside the bracket • Divide each term by the factor to get the new terms inside the bracket • Always check by expanding your bracket	$2x + 4y$ $5x^2y - 10xy$	$2(x + 2y)$ $5xy(x - 2)$
Expressions			
30.	Linear	Can be represented by a straight line No indices above 1	e.g. $2x + 2$
31.	Quadratic	An expression where the highest index is 2	e.g. $2x^2 + 2x + 2$

Expanding double brackets

32.	Everything in the first bracket must be multiplied by everything in the second

33.

Grid method

$(x+4)(x+7)$

$\times$	x	$+4$
x	x^2	$4x$
$+7$	$7x$	28

$= x^2 + \underbrace{4x + 7x} + 28$

$= \underline{\underline{x^2 + 11x + 28}}$

FOIL method

FIRST :	$(x+3)(x-4)$	gives	$x \times x = x^2$
OUTER :	$(x+3)(x-4)$	gives	$x \times (-4) = -4x$
INNER :	$(x+3)(x-4)$	gives	$3 \times x = 3x$
LAST :	$(x+3)(x-4)$	gives	$3 \times (-4) = -12$

Factorising a quadratic expression

34.	Factorising a quadratic in the form of $ax^2 + bx + c$	Multiply to 5 Factorise $x^2 + 5x + 6$ ← Add to 6 2 and 3 add to 5 2 and 3 multiply to 6 $(x+2)(x+3)$ Check: $(x+2)(x+3) = x^2 + 5x + 6$
35.	Difference of two squares	A special type of quadratic which only has two terms. One term is subtracted from the other $x^2 - 25 = x^2 - 5^2 = (x+5)(x-5)$ $y^2 - 49 = y^2 - 7^2 = (y+7)(y-7)$ $a^2 - 16 = a^2 - 4^2 = (a+4)(a-4)$

Equations

36.	To solve equations we need to use inverse operations
37.	What ever you do to one side of the equals sign you must do the same to the other

38.	One step	$x + 4 = 7$ $(-4) \quad (-4)$ $x = 11$ $x - 5 = 12$ $(+5) \quad (+5)$ $x = 17$ $3x = 18$ $(\div 3) \quad (\div 3)$ $x = 1$ $\frac{x}{4} = 6$ $(\times 4) \quad (\times 4)$ $x = 24$	
39.	Two step	Requires the use of two inverse operations	$2x - 7 = 19$ $2x = 26$ $x = 13$
40.	With brackets	Expand the brackets first $5(2x + 1) = 35$ $10x + 5 = 35$ $10x = 30$ $x = 3$	OR if possible divide by the number outside of the bracket first $4(2x + 4) = 20$ $2x + 4 = 5$ $2x = 1$ $x = \frac{1}{2}$
41.	Unknowns on both sides	Start by eliminating the unknown from one of the signs.	$5x + 2 = 3x - 8$ $2x + 2 = -8$ $2x = -10$ $x = -5$
42.	With fractions	Eliminate any terms that are being added or subtracted separate from the fraction first. $\frac{f}{5} + 2 = 8$ $\frac{f}{5} = 6$ $f = 30$	If everything is part of the fraction then multiply by the denominator first. $\frac{f+2}{5} = 8$ $f + 2 = 40$ $f = 38$

Changing the subject of a formula (rearranging)

43.	Always use inverse operations to isolate the term you have been asked to make the subject		
	If the letter you want as the subject appears twice you will need to factorise		
	Make u the subject: $v = u + at$ $(-\boldsymbol{at})$ $v - at = u$ So $u = v - at$	Make u the subject: $v^2 = u^2 + 2as$ $(-\boldsymbol{2as})$ $v^2 - 2as = u^2$ $(\sqrt{\quad})$ $\sqrt{v^2 - 2as} = u$ So $u = \sqrt{v^2 - 2as}$	Make m the subject: $I = mv - mu$ $(\boldsymbol{Factorise})$ $I = m(v - u)$ $(\div (\boldsymbol{v - u}))$ $\frac{I}{v - u} = m$ So $m = \frac{I}{v - u}$

Iteration

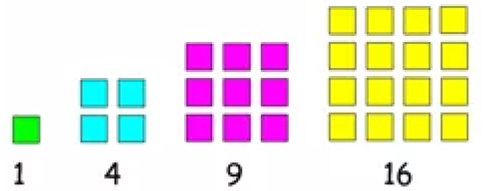
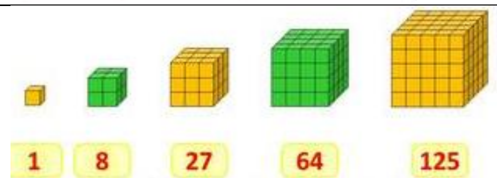
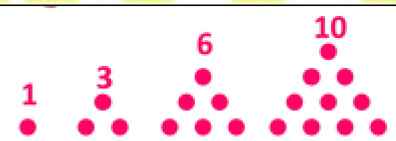
44.	Iteration	The act of repeating a process to generate a sequence of outcomes or with the aim of of appraoching a desired result e.g. finding a solution to an equation
45.	Iterative sequence	The relationship between consecutive terms
46.	Roots	Solutions to an equation
47.	Change of sign	Two values with a root between them

Sequences

48.	Sequence	An order pattern of numbers or diagrams
49.	Term	One of the numbers or diagrams in a sequence
50.	Term to term rule	The rule for moving from one term to the next in a sequence
51.	Formula	A rule written to describe a realtionship between twp quantities
52.	Arithmetic sequence	A sequence where the term to term rule is to addd or subtract the same amount each time
53.	Quadratic sequence	A sequence where the term to term rule is changing by the same amount each time The second difference is a constant amount.
54.	Geometric sequence	A sequence where the term to term rule is to multiply by the same amount each time
55.	Common ratio	The value a geometric sequence is multiplied by from one term to the next Denoted by the letter r
56.	Series	The sum of the terms in a sequence
57.	Position to term rule	The rule for finding any value of a sequence
58.	nth term rule for an arithmetic sequence	The rule to find any term in a sequence of numbers • Find the common difference between the terms • This becomes you coefficient of n (this is the times table the sequenc is linked to) • The number you need to add or subtract to get to the second term becomes the second term in the nth term rule 
59.	Nth term for a quadratic sequence	• Find the first difference • Find the second difference • Halve the second difference and multiply by n^2 to gain a new sequence of an^2 • Generate the first few term sof this seuence then subtract from the original sequence

		• Find the nth term of the remianing sequence $bn + c$ • The entire nth term is then $an^2 + bn + c$
60.	nth term for a geometric sequence	• Divide the second sequence by the first to find the common ratio, r • The nth term is ar^{n-1} where a is the first term and n is the term position in the sequence
61.	Finite	Has a final point
62.	Infinite	Carries on forever
63.	Ascending	Increases
64.	Descending	Decreases
65.	Linear function	An aruthmetic sequence that can be represented by a straight line graph

Special Sequences

66.	Square numbers	1, 4, 9, 16, 25, 36, 49, 64, 81, 100	
67.	Cube numbers	1, 8, 27, 64, 125	
68.	Triangular numbers	1, 3, 6, 10, 15, 21, 28	
69.	Fibonacci sequence	A sequence where each term is the sum of the two previous terms e.g. 1, 1, 2, 3, 5, 8, 13, 21...	

GCSE Mathematics Higher Unit 3

Definitions

1.	Qualitative Data	Non-numerical data	i.e. Colour of car
2.	Quantitative Data	Numerical data	i.e. House number
3.	Discrete Data	Numerical data that <u>CANNOT</u> be shown in decimals	i.e. Number of children in a class
4.	Continuous Data	Numerical data that <u>CAN</u> be shown in decimals	i.e. The heights of children in a class
5.	Grouped Data	Numerical data given in intervals	i.e. Year group ranges: Year 7-9 Year 10-11 Year 12-13

Averages

6.	Measure of location	A single value that describes a position in a data set	
7.	Measure of central tendency	A single value that describes the centre of the data	
8.	Measure of spread	A measure of how spread out the data is	
		Also known as 'measures or dispersion' or 'measures of variation'	
		Two simple measures of spread are range and interquartile range (IQR)	
9.	Mode (modal class)	The value that occurs most often	
10.	Range	The difference between the largest and smallest values in the data set	
11.	Median	The middle value when the data values are put in ascending order	
12.	Mean	Found by adding all number sin the data set and dividing by the number of values in the set	
		Can be calculate using the formula $\bar{x} = \frac{\Sigma x}{n}$	Where: $\bar{x}$ is the mean Σx is the sum of the data values n is the number of data values
		Mean from a frequency table $$\bar{x} = \frac{\Sigma f x}{\Sigma f}$$ Where $\Sigma f x$ is the sum of the products of data values and their frequencies and Σf is the sum of the frequencies	

Advantages and disadvantages of averages

	Average	Advantages	Disadvantages
13.	*Mean*	Every value makes a difference	Affected by extreme values
	Median	Not affected by extreme values	May not change even if a data value changes
	Mode	Easy to find; not affected by extreme values; can be used for non-numerical data	There may not be a mode

Averages from frequency tables

14.	Modal class	The class with the highest frequency
15.	Median	If the total frequency is n, then the median lies in the class with the $\frac{n+1}{2}$th value in it.

16. Mean from a frequency table

Times →
Add ↓↓
Divide ←

No of make-up items in handbags

No of Items x	Freq f	f x x
1	7	1 x 7 = 7
2	2	2 x 2 = 4
3	1	3 x 1 = 3
4	4	4 x 4 = 16
5	2	5 x 2 = 10
	16	40

$$\text{Mean} = \frac{40}{16} = 2.5$$

17. Estimated mean from a grouped frequency table

Times →
Add ↓↓
Divide ←

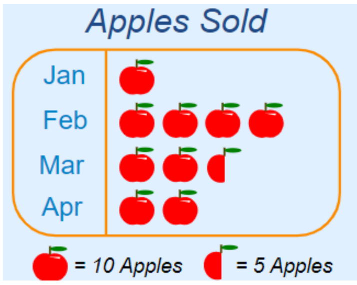
Class Interval	*Mid-point*	*Frequency*	*Mid-point × Frequency*
$140 \le h < 150$	145	6	$145 \times 6 = 870$
$150 \le h < 160$	155	16	$155 \times 16 = 2480$
$160 \le h < 170$	165	21	$165 \times 21 = 3465$
$170 \le h < 180$	175	8	$175 \times 8 = 1400$
	Totals	**51**	**8215**

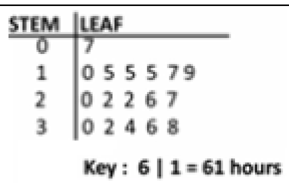
Mean = 8215 ÷ 51
=161.07843...
= 161.08 (2dp)

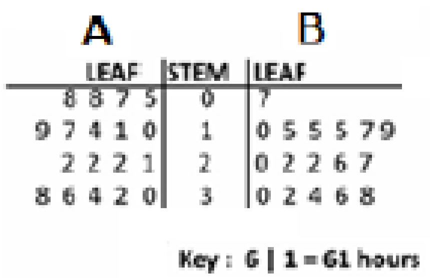
18.	Estimate of range from grouped frequency table	The maxiumum possible value minus the smallest possible value.

Averages from charts/graphs

<table>
<tr>
<td>19.</td>
<td>Bar chart</td>
<td>A chart to display discrete data where the height of the bar shows the frequency.

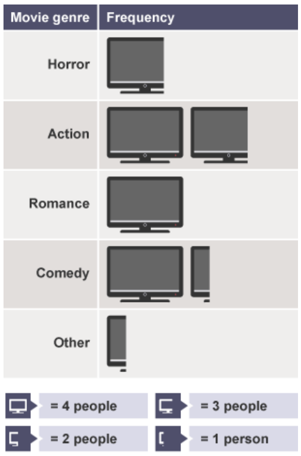
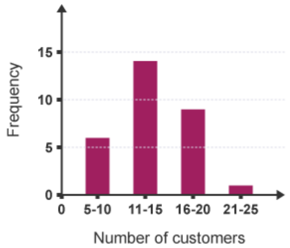
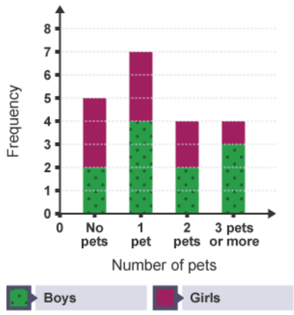
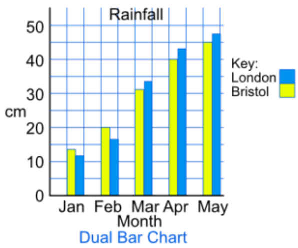
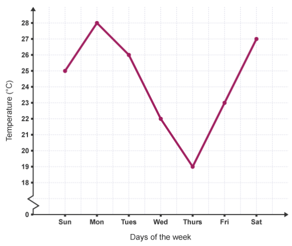

</td>
<td>Mean: 23 ÷ 10 = 2.3
Median: 2.5
Mode : 3
Range: 4-1 = 3</td>
</tr>
<tr>
<td>20.</td>
<td>Pictogram</td>
<td>A chart that uses pictures to represent quantities. Must include a key.


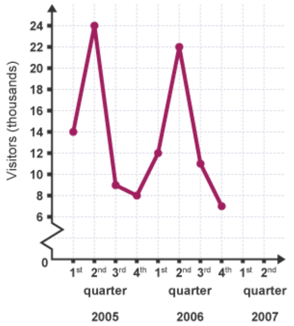
</td>
<td>Mean: 95÷4 = 23.75
Median: 22.5
Range: 30</td>
</tr>
<tr>
<td>21.</td>
<td>Stem and leaf diagram</td>
<td>


A diagram that shows groups of data arranged by place value. ‘Leaves’ should be in order. Must have a key.</td>
<td>Mean: 385÷17 = 22.6
Median: 22
Mode: 15
Range: 38-7 = 31</td>
</tr>
<tr>
<td>22.</td>
<td>Back to back stem and leaf</td>
<td>Compares two sets of results. Must have a key.


</td>
<td>Set A
Mean: 356÷18 = 19.8
Median: 20
Mode: 22
Range: 38-5 = 33

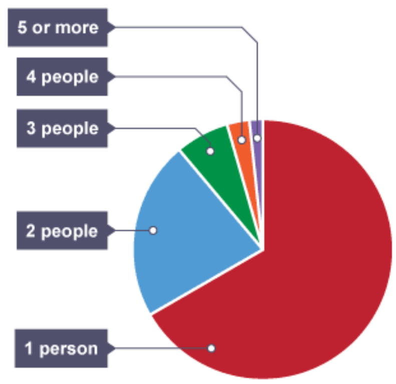
Set B
Mean: 385÷17 = 22.6
Median: 22
Mode: 15
Range: 38-7 = 31</td>
</tr>
<tr>
<td colspan="4"><h2>Representing data</h2></td>
</tr>
<tr>
<td>23.</td>
<td>Two-Way Tables</td>
<td>
<table>
<tr><td></td><td>Boys</td><td>Girls</td><td>TOTAL</td></tr>
<tr><td>Pet</td><td>9</td><td>4</td><td>13</td></tr>
<tr><td>No Pet</td><td>2</td><td>5</td><td>7</td></tr>
<tr><td>TOTAL</td><td>11</td><td>9</td><td>20</td></tr>
</table>
</td>
<td>Two-way tables are a way of sorting data with two categories.</td>
</tr>
</table>

24.	Pictograms		Used to show frequencies Pictures and images used to represent frequency A key at the bottom helps you interpret the diagram
25.	Bar Charts		Frequency on the vertical axis, and categories along the horizontal axis. Used to compare frequencies
26.	Composite Bar Chart		Frequency on the vertical axis, and categories along the horizontal axis. Two shades used to show difference in proportion between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
27.	Comparative Bar Chart		Frequency on the vertical axis, and categories along the horizontal axis. Bars are next to each other and used to show difference in frequency between sub-groups (i.e. gender) Used to compare frequencies within sub-groups
28.	Line Graph		A line graph is used to show a change or relationship between two variables. Once the points are plotted, they are joined with straight lines.

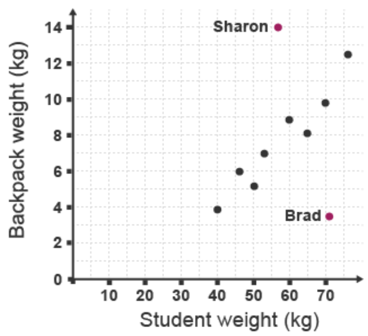
<table>
<tr><td>29.</td><td>Time-Series</td><td>
</td><td>A time-series graph plots frequencies (vertical) axis against time (horizontal).

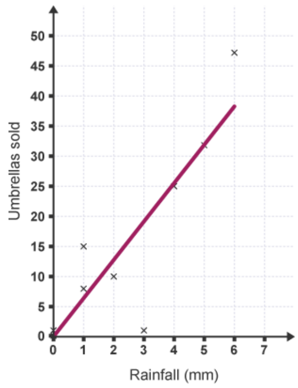
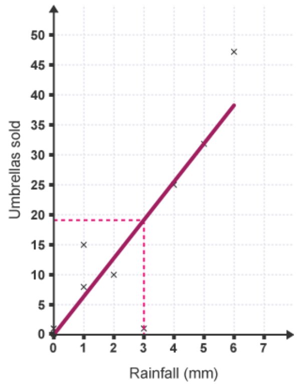
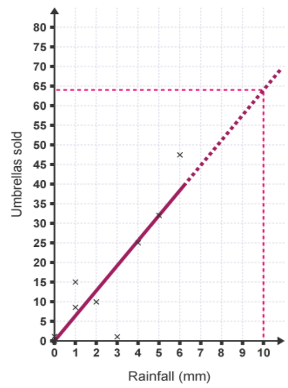
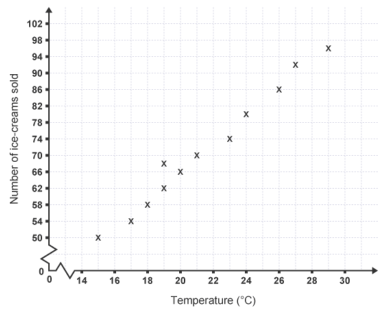
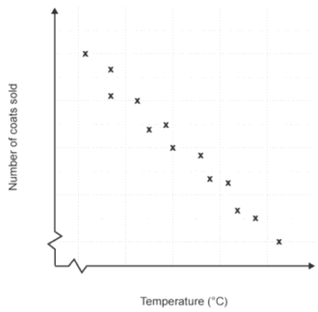
It is used to spot trends over time.

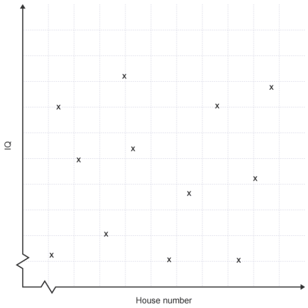
Time could be: weeks, months, quarters (3 months), years</td></tr>
<tr><td>30.</td><td>Stem & Leaf Diagrams:</td><td>
<table>
<tr><td></td><td>Key: 1 | 1 = 11 marks</td></tr>
<tr><td>0</td><td>9</td></tr>
<tr><td>1</td><td>1 6 7 8</td></tr>
<tr><td>2</td><td>1 2 7 7 8 8 9</td></tr>
<tr><td>3</td><td>0 0 1 5 6 7 8 9</td></tr>
<tr><td>4</td><td>0 1 2 5</td></tr>
</table></td><td>A stem and leaf diagram shows numbers in a table format. It can be a useful way to organise data to find the median, mode and range of a set of data.

Only one digit is allowed to be a 'leaf'
There should be a key to help you interpret the diagram</td></tr>
<tr><td>31.</td><td>Pie Charts</td><td colspan="2">A pie chart is a chart represented by a circle. It shows the proportion of each group at a glance.
<table>
<tr><th>People travelling in a vehicle</th><th>Frequency</th><th>Calculation</th><th>Angle</th></tr>
<tr><td>1 person</td><td>120</td><td>$\frac{120}{180} \times 360°$</td><td>240°</td></tr>
<tr><td>2 people</td><td>40</td><td>$\frac{40}{180} \times 360°$</td><td>80°</td></tr>
<tr><td>3 people</td><td>13</td><td>$\frac{13}{180} \times 360°$</td><td>24°</td></tr>
<tr><td>4 people</td><td>5</td><td>$\frac{5}{180} \times 360°$</td><td>10°</td></tr>
<tr><td>5 or more</td><td>2</td><td>$\frac{2}{180} \times 360°$</td><td>4°</td></tr>
<tr><td>Total</td><td>180</td><td></td><td></td></tr>
</table>

</td></tr>
</table>

Scatter Graphs

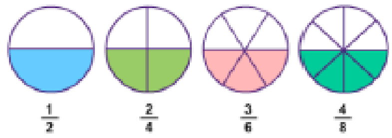
<table>
<tr><td>32.</td><td>Outliers</td><td>
</td><td>Outliers don't follow the trend</td></tr>
</table>

33.	Line of Best Fit		A sensible straight line that goes as centrally as possible through the points plotted. It should also follow the same steepness of the crosses.	
34.	Interpolate		Using a line of best fit to estimate data <u>WITHIN</u> our range <u>For example</u>: To estimate how many umbrellas are sold with 3mm rain. • Find where 3 mm of rainfall is on the graph. • Draw a line by going across from 3 mm and then down.	
35.	Extrapolate		Continuing a line of best fit to estimate data <u>BEYOND</u> our range (not as reliable as interpolation) <u>For example</u>: To estimate how many umbrellas are sold with 10mm rain. • Continue the line of best fit. • Find where 10mm of rainfall is on the graph. • Draw a line by going across from 10mm and then down.	
36.	Positive Correlation		BOTH variables increase with each other	i.e. Ice creams sold vs Temperature
37.	Negative Correlation		ONE variable increases as the other decreases	i.e. Coats sold vs temperature

38.	No Correlation		NO relationship between variables	i.e. IQ and House Number
39.	Causation	If one variable causes a change in the other. • i.e. an increase temperature WILL cause an increase ice cream sales • i.e. the number of bee stings WILL NOT cause an increase in ice cream sales (although both will increase in hot weather)		

GCSE Mathematics Higher Unit 4

Fractions

1.	Fraction	Part of a whole	
2.	Numerator	The number on the top of the fraction	$\frac{numerator}{denominator}$
3.	Denominator	The number on the bottom of the fraction	
4.	Equivalent fractions	Fractions that have the same value but look different.	
5.	Improper fraction	A fraction where the numerator is larger than the denominator.	e.g. $\frac{4}{3}$
6.	Mixed number	A number made from integer and fraction parts.	e.g. $2\frac{2}{3}$
7.	Unit fraction	A fraction that has a numerator of 1	
8.	Reciprocal	The reciprocal of a number is 1 divided by the number.	e.g. the reciprocal of 3 is $\frac{1}{3}$
		Dividing by a number is the same as multiplying by its reciprocal	e.g. $\times$ by $\frac{1}{3}$ is the same as $\div$ by 3

Fractions - processes

9.	Simplifying fractions	Divide the numerator and denominator by the HCF.	$\frac{24}{30} = \frac{4}{5}$
10.	Finding equivalent fractions	Multiply the numerator and denominator by the same number	$\frac{4 \times 2 = 8}{8 \times 2 = 16}$
11.	Comparing fractions	Write them with a common denominator	
12.	Fraction of an amount	Amount divided by the denominator then multiplied by the numerator	e.g. $\frac{5}{7}$ of 42 $42 \div 7 \times 5 = 30$
13.	Multiply fractions	Multiply the numerators and multiply the denominators	$\frac{6}{7} \times \frac{4}{5} = \frac{6 \times 4}{7 \times 5} = \frac{24}{35}$
14.	Divide fractions	• Flip the second fraction (find the reciprocal). • Change the divide to multiply. • Multiply the fractions.	$\frac{4}{7} \div \frac{5}{6} = \frac{4}{7} \times \frac{6}{5} = \frac{4 \times 6}{7 \times 5} = \frac{24}{35}$
15.	Add or subtract fractions	• Write as fractions with a common denominator. • Add or subtract the numerators	$\frac{2}{8} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$

16.	Convert improper fractions to mixed numbers	• Divide the numerator by the denominator • The answer gives the whole number part. • The remainder becomes the numerator of the fraction part with the same denominator.	$\frac{43}{6} = 7\frac{1}{6}$
17.	Convert mixed numbers to improper fractions	• Multiply the denominator by the whole number part. • Add the numerator to this. • Put the answer to this back over the denominator	$7\frac{1}{6} = \frac{6 \times 7 + 1}{6} = \frac{43}{6}$
18.	Adding and subtracting mixed numbers	• Convert mixed numbers to improper fractions • Transform both fractions so they have the same denominator • Add or subtract the numerators Convert back to mixed number if applicable	
19.	Multiplying mixed numbers	• Convert mixed numbers to improper fractions • Multiply numerators and multiply the denominators Convert back to mixed number if applicable	
20.	Dividing mixed numbers	• Convert mixed numbers to improper fractions • Flip the second fraction (find the reciprocal) • Change the divide sign to a multiply • Multiply the fractions Convert back to mixed number if applicable	
Percentages			
21.	Percentage	Means 'out of 100'	
22.	Multiplier	A decimal you multiply by to represent a percentage	
		To use a multiplier to find a percentage, divide your percentage by 100, then multiply the amount by this value.	
23.	Percentage increase	Calculate the percentage and add onto the original	
		Or use a multiplier	$amount \times \frac{100 + \%\ increase}{100}$
24.	Percentage decrease	Calculate the percentage and subtract from the original	
		Or use a multiplier	$amount \times \frac{100 - \ \%\ increase}{100}$
25.	Percentage change	$\frac{Change}{Original} \times 100$	
26.	Express one number as a percentage of another	$\frac{Number\ 1}{Number\ 2} \times 100$	

<table>
<tr><td rowspan="2">27.</td><td rowspan="2">Reverse percentage</td><td colspan="2">Use when asked to find the priginal amount after a percentage increase or decrease.</td></tr>
<tr><td colspan="2">$$\text{Original Value} \times \text{Multiplier} = \text{New Value}$$ $$\text{Original Value} = \frac{\text{New Value}}{\text{Multiplier}}$$</td></tr>
<tr><td>28.</td><td>Interest</td><td colspan="2">A fee paid for borrowing money or money earnt through investing.</td></tr>
<tr><td>29.</td><td>Simple interest</td><td>Interest that is calculated as a percentage of the original</td><td>I = Prt

I – Interest
P – Original amount
r – interest rate
t - time</td></tr>
<tr><td rowspan="2">30.</td><td rowspan="2">Compound interest</td><td>When interest is calculate on the original amount and any previous interest</td><td rowspan="2">$P\left(1+\frac{R}{100}\right)^n$

P – Original amount
R – Interest rate
n – the number of interest periods (e.g. yrs)</td></tr>
<tr><td>OR

$Original \times Multiplier^{time}$</td></tr>
<tr><td>31.</td><td>Tax</td><td colspan="2">A financial charge placed on sales or savings by the government e.g. VAT</td></tr>
<tr><td>32.</td><td>Loss</td><td colspan="2">Income minus all expenses, resulting in a negative value</td></tr>
<tr><td>33.</td><td>Profit</td><td colspan="2">Income minus all expenses, resulting in a positive value</td></tr>
<tr><td>34.</td><td>Depreciation</td><td colspan="2">A reduction in the value of a product over time</td></tr>
<tr><td>35.</td><td>Annual</td><td colspan="2">Means yearly</td></tr>
<tr><td>36.</td><td>Per annum</td><td colspan="2">Means per year</td></tr>
<tr><td>37.</td><td>Salary</td><td colspan="2">A fixed regular payment, often paid monthly</td></tr>
<tr><td colspan="4">FDP Conversions</td></tr>
<tr><td>38.</td><td>Percentage to decimal</td><td colspan="2">Divide by 100</td></tr>
<tr><td>39.</td><td>Decimal to percentage</td><td colspan="2">Multiply by 100</td></tr>
<tr><td>40.</td><td>Fraction to percentage</td><td colspan="2">Find an equivalent fraction with 100 as the denominator</td></tr>
<tr><td>41.</td><td>Percentage to fraction</td><td colspan="2">Write as a fraction over 100 then simplify</td></tr>
<tr><td>42.</td><td>Fraction to decimal</td><td colspan="2">Carry out division or convert to a percentage first</td></tr>
<tr><td>43.</td><td>Decimal to fraction</td><td colspan="2">Use place value to find the denominator and simplify or convert to a percentage first</td></tr>
</table>

Basics to memorise

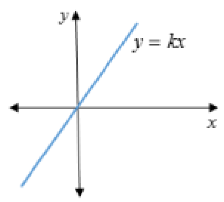
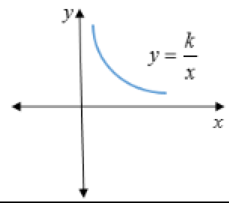
44.	Fraction	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
	Decimal	0.01	0.1	0.125	0.2	0.25	$0.\dot{3}$	0.5	$0.\dot{6}$	0.75
	Percentage	1%	10%	12.5%	20%	25%	$33.\dot{3}\%$	50%	$66.\dot{7}\%$	75%

Terminating and recurring decimals

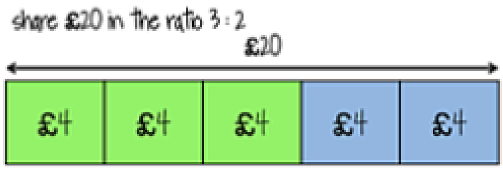
45.	Terminating decimal	Decimals that can be written exactly	e.g. 0.38
46.	Recurring decimal	Decimals where one digit or groups of digits are repeated	e.g. $0.\dot{7}$ = 0.7777... $0.\dot{8}5\dot{3}$ = 0.853853...
47.	Converting a recurring decimal to a fraction	1. Let x = recurring decimal. 2. Let n = the number of recurring digits. 3. Multiply the recurring decimal by 10^n. 4. Subtract (1) from (3) to eliminate the recurring part. 5. Solve for x, expressing your answer as a fraction in its simplest form. $0.\dot{7}$ (one recurring digit) $x = 0.7777...$ $10x = 7.777...$ $10x - x = 7$ $9x = 7$ $x = \frac{7}{9}$	$1.2\dot{5}\dot{6}$ (two recurring digits) $x = 1.25656...$ $100x = 125.6565...$ $100x - x = 125.6565... - 1.256565...$ $99x = 124.4$ $x = \frac{124.4}{99} = \frac{1244}{990} = \frac{622}{495}$
48.	Converting a fraction to recurring decimals	Carry out the neccesary division using a calcualtor or bus stop division	e.g. $\frac{4}{7}$ means 4 ÷ 7 0. 5 7 1 4 2 8 5 7 7 | 4.^{4}0^{5}0^{1}0^{3}0^{2}0^{6}0^{4}0^{5}0

Ratio and Proportion

49.	Ratio	A relationship between two or more quantities
50.	Unit ratio	Used to compare ratios, one of the parts is 1
		The only time it is permissible to have a decimal in a ratio
51.	Equivalent	Ratios that have the same simplified form are said to be equivalent

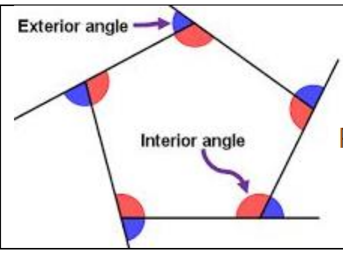
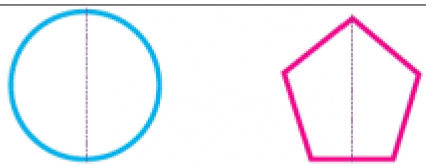
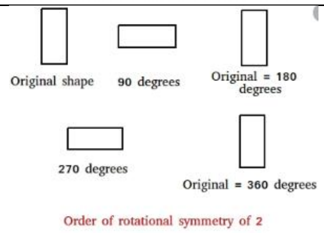
52.	Scale	A ratio that represents the relationship between a length on a drawing or a map and the actual length	
53.	Proportion	Compares a part with a whole	
54.	Direct proportion	Two quantities increase at the same rate	$y \propto x$ $y = kx$ for a constant k 
		Graph is a straight line that goes through the origin	
55.	Inverse/indirect proportion	One variable increases at a constant rate as the second variable decreases	$y \propto \frac{1}{x}$ $y = \frac{k}{x}$ for a constant k 
56.	Proportional	A change in one is always accompanied by a change in the other	
57.	Constant of proportionality	Represented by k	
		Its value stays the same	
58.	Share	Splitting into parts as defined by a ratio	
59.	Unitary method	Finding the value of 1 item then using this to find the value of any number of that item	
		Use to work out which products give the best value for money	

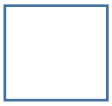
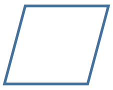
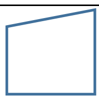
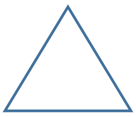
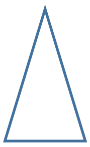
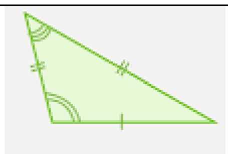
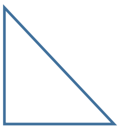
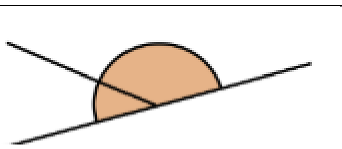
Working with ratios

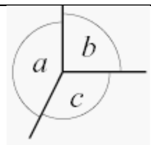
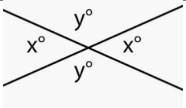
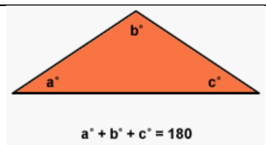
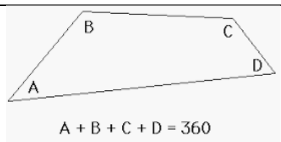
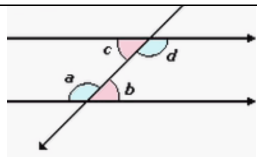
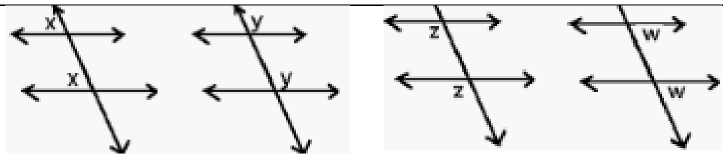
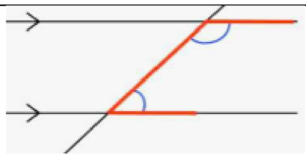
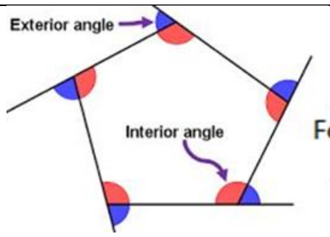
60.	Simplifying ratio	Divide all parts by the highest common factor	e.g. 12:4 simplifies to 3:1 (divided by HCF of 4)
		All parts in the simplified version must be integers	
61.	Divide in a given ratio	Divide an amount so the ratio of the final values simplifies to the given ratio	

GCSE Mathematics
Higher
Unit 5

Shapes and angles - definitions

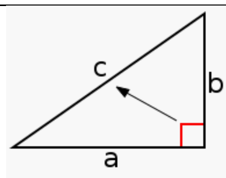
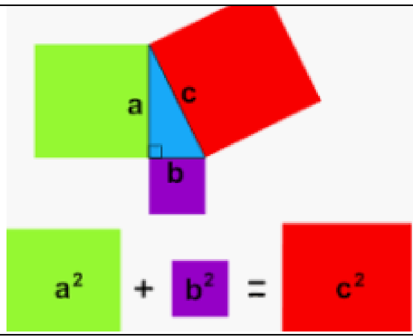
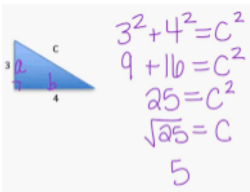
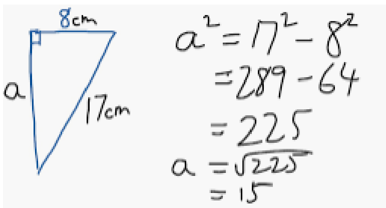
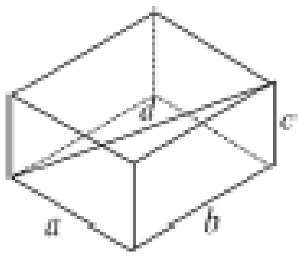
1.	Angle	A measure of turn, measured in degrees ◦	
2.	Protractor	Instrument used to measure the size of an angle	
3.	Acute angle	An angle less than 90◦	
4.	Right angle	A 90◦ angle	
5.	Obtuse angle	An angle more than 90◦ but less than 180◦	
6.	Reflex angle	An angle more than 180◦	
7.	Parallel lines	Lines that are equal distance apart that will never meet even when extended	
8.	Perpendicular lines	Lines that intersect at a right angle	
9.	Polygon	A 2D shape with straight lines only	
10.	Regular polygon	A polygon where: All sides are the same length All angles are the same size	
11.	Interior angles (I)	An angle inside a polygon	 For any polygon: $I + E = 180^0$
12.	Exterior angles (E)	An angle outside a polygon	
13.	Congruent	Shapes that are the same shapes and size, they are identical.	
14.	Similar	Shapes that are the same shape but are different sizes	
15.	Bisect	Cut in half	
16.	Tessellate	Fit together without leaving gaps	
17.	Symmetry	A shape has symmetry if a central line is drawn to show both sides are exactly the same. We call these lines of symmetry	
18.	Rotational symmetry	A shape has rotational symmetry when it looks the same after some rotation of less than a full turn	

Quadrilaterals (4 sided shapes)				
19.	Square		4 equal sides 4 equal angles 2 pairs of parallel sides Diagonals cross at right angles	4 lines symmetry Rotational symmetry order 4
20.	Rectangle		2 pairs of equal sides 4 right angles 3 pairs of parallel sides	2 lines of symmetry Rotational symmetry order 2
21.	Rhombus		4 equal sides 2 pairs of equal angles 2 pairs of parallel sides Diagonals cross at right angles	2 lines of symmetry Rotational symmetry order 2
22.	Parallelogram		2 pairs of equal sides 2 pairs of equal angles 2 pairs of parallel sides	0 lines of symmetry Rotational symmetry order 2
23.	Kite		2 pairs of equal sides 1 pair of equal angles 2 pairs of parallel sides Diagonals cross at right angles	1 line of symmetry Rotational symmetry order 1
24.	Trapezium		One pair of parallel lines	
25.	Isosceles trapezium		1 pair of parallel sides 1 pair of equal sides 2 pairs of equal angles	1 line of symmetry Rotational symmetry order 1
Triangles (3 sided shapes)				
26.	Equilateral		3 equal sides 3 equal angles	3 lines of symmetry Rotational symmetry order 3
27.	Isosceles		2 equal sides 2 equal angles	1 line of symmetry Rotational symmetry order 1
28.	Scalene		No equal sides No equal angles	
29.	Right-angled		1 right angle Can be scalene or isosceles	
Basic angle rules				
30.	Angles on a straight line add to 180°			

31.	Angles around a point add up to 360◦	
32.	Vertically opposite angles are equal	
33.	Angles in a triangle add to 180◦	
34.	Angles in a quadrilateral add up to 360◦	
Angles on parallel lines		
35.	Alternate angles are equal	
36.	Corresponding angles are equal	
37.	Co-interior angles add up to 180◦	
Angles in polygons		
38.	Interior and exterior angles add to give 180◦	 For any polygon: $I + E = 180^0$
39.	Sum of interior angles	For a 'n' sided polygon Sum of interior angles = 180 x (n-2)

40.	Size of one interior angle	For a 'n' sided polygon Interior angle = $\frac{180\ x\ (n-2)}{n}$
41.	Sum of exterior angles	For all polygons, sum of exterior angles = 360°
42.	Regular polygons	Exterior angle = 360 ÷ number of sides
		Number of sides = 360 ÷ exterior angle
		Interior angle = 180 – exterior angle

Pythagoras' Theorem

43.	Hypotenuse	The longest side of a right-angled triangle	
		It is always opposite the right angle	
44.	Right- angled triangle	A triangle that contains a right angle	
45.	Pythagoras' Theorem	$a^2 + b^2 = c^2$	
		Where c is the hypotenuse	
		Where a and b are the two shorter sides	
46.	To find the hypotenuse (c)		• Square • Add • Square root
47.	To find a short side (a/b)		• Square • Subtract • Square root
48.	Pythagoras' in 3D	$a^2 + b^2 + c^2 = d^2$	
		$d^2 - b^2 - c^2 = a^2$	

Trigonometry - Right angled – SOH CAH TOA

<table>
<tr><td>49.</td><td>Trigonometry</td><td colspan="3">The ratios between the sides and angles of triangles</td></tr>
<tr><td>50.</td><td>Labelling the triangle</td><td colspan="2">

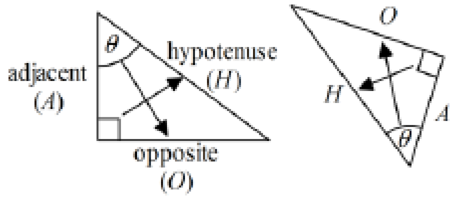
θ is the angle involved

H is the hypotenuse

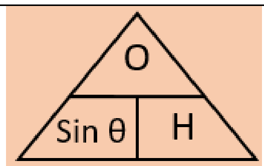
O is the opposite

A is the adjacent

</td><td>



</td></tr>
<tr><td>51.</td><td>Sine</td><td>SOH</td><td>

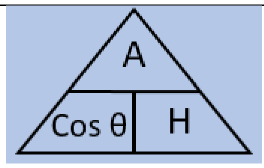


</td><td>

$Sin\ \theta = \frac{Opp}{Hyp}$

$\theta = Sin^{-1}\frac{Opp}{Hyp}$

</td></tr>
<tr><td>52.</td><td>Cosine</td><td>CAH</td><td>

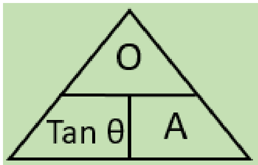


</td><td>

$Cos\ \theta = \frac{Adj}{Hyp}$

$\theta = Cos^{-1}\frac{Adj}{Hyp}$

</td></tr>
<tr><td>53.</td><td>Tangent</td><td>TOA</td><td>



</td><td>

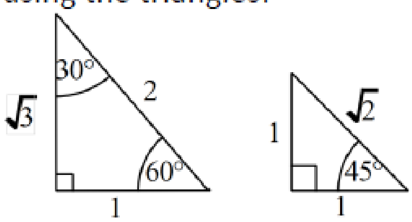
$Tan\ \theta = \frac{Opp}{Adj}$

$\theta = Tan^{-1}\frac{Opp}{Adj}$

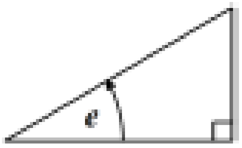
</td></tr>
<tr><td>54.</td><td>Exact Values</td><td colspan="3">

<table>
<tr><td>θ</td><td>0°</td><td>30°</td><td>45°</td><td>60°</td><td>90°</td></tr>
<tr><td>Sin θ</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>Cos θ</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>Tan θ</td><td>0</td><td>$\frac{\sqrt{3}}{3}$</td><td>1</td><td>$\sqrt{3}$</td><td></td></tr>
</table>

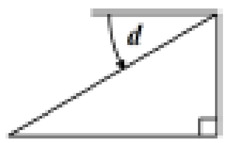
These can be found using the triangles:



</td></tr>
<tr><td>55.</td><td>Angle of elevation</td><td>



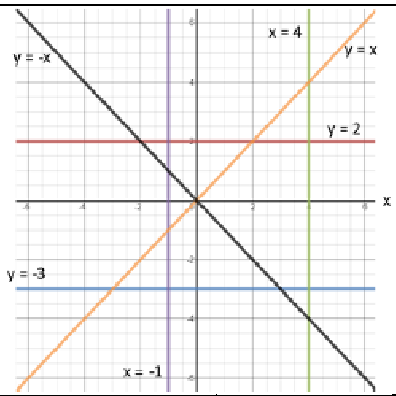
</td><td>Angle of depression</td><td>



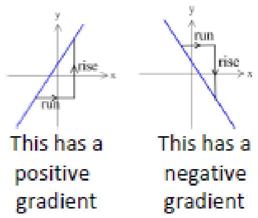
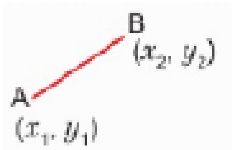
</td></tr>
</table>

GCSE Mathematics
Higher
Unit 6

Graphs - definitions

1.	Axis	A reference line on a graph	
2.	Axes	Plural of axis	
3.	Quadrant	A quarter of a graph separated by a axes	
4.	Coordinate	Used to show a position on a coordinate plane, (x, y)	
		First coordinate is the horizontal position, (x axis) and the second is the vertical position (y axis)	
5.	Origin	The point (0,0) on a set of axes	
6.	Plot	Mark a position or positions on a graph	
7.	y intercept	The y value where a graph crosses the y axis	where x=0
8.	x intercept	The x value where a graph crosses the x axis	where y=0
9.	Parallel	Lines that are equal distance apart that if extended will never meet	
10.	"y=" graph	Constant y coordinate	
		Will be parallel to the x axis	
11.	"x=" graph	Constant x coordinate	
		Will be parallel to the y axis	
12.	Linear function	An arithmetic sequence that can be represented by a straight line graph	
13.	Linear equation	An equation that produces a straight line graph	
14.	Equation of a line	$y = mx + c$ $m = gradient$ $c = y\ intercept$	$ax + by + c = 0$ Where a, b and c are integers

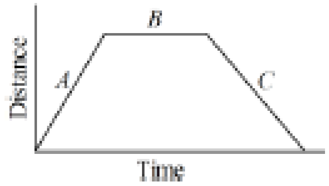
Coordinate geometry

15.	Gradient	The steepness of a graph $$Gradient = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$$	
16.	Gradient between two points	If A = (x_1, y_1) and B = (x_2, y_2) The gradient of line AB = $\frac{y_2-y_1}{x_2-x_1}$	
17.	Parallel lines	Have the same gradients	
18.	Perpendicular	Lines that are at right angles to one another Lines that are perpendicular are the negative reciprocal of one another If two lines are perpendicular, the product of their two gradients is -1	If a line has a gradient of m, the gradient of a line perpendicular to it will have a gradient of $-\frac{1}{m}$
19.	Mid-point	The coordinate half way between two point	If A = (x_1, y_1) and B = (x_2, y_2) the mid-point is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
20.	Distance between two points	Distance (d) between $(x_{1,}y_1)$ and $(x_{2,}y_2)$ can be found using the formula $$d = \sqrt{(x_2 - x_{1,})^2 + (y_2 - y_{1,})^2}$$	

Real life graphs

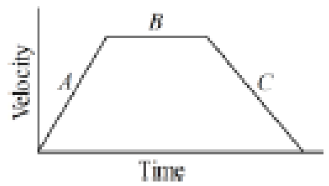
21.	Steady speed	Travelling the same distance each minute
22.	Velocity	Speed in a particular direction
23.	Rate of change	Shows how a variable changes over time
24.	Acceleration	How fast velocity changes; measured in m/s^2 or km/s^2 etc

Distance - Time graphs

<table>
<tr><td>25.</td><td>Represent a journey</td><td rowspan="7">

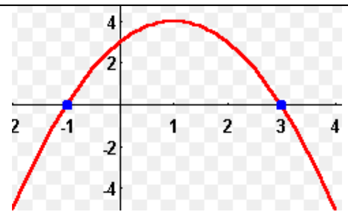
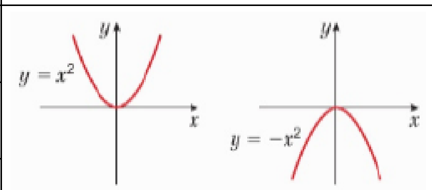
A = steady speed,
B = no movement,
C = steady speed back to start</td></tr>
<tr><td>26.</td><td>Vertical axis represents the distance from the starting point</td></tr>
<tr><td>27.</td><td>Horizontal axis represents the time taken</td></tr>
<tr><td>28.</td><td>Straight lines mean constant speed</td></tr>
<tr><td>29.</td><td>Horizontal lines mean no movement</td></tr>
<tr><td>30..</td><td>Gradient = speed</td></tr>
<tr><td>31.</td><td>Average speed = = $\frac{total\ distance}{total\ time}$</td></tr>
</table>

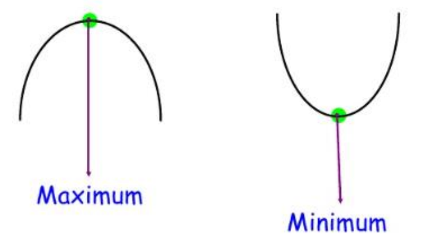
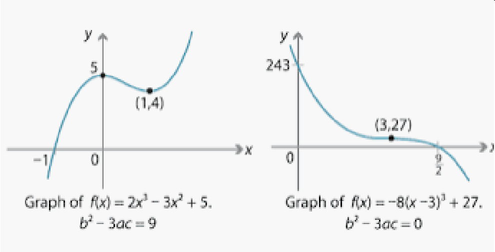
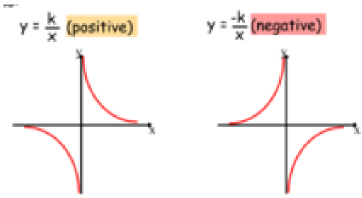
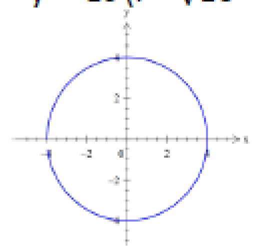
Velocity – Time graphs

<table>
<tr><td>32.</td><td>Represents the speed at given times</td><td rowspan="6">

A = steady acceleration,
B = constant speed,
C = steady deceleration back to a stop</td></tr>
<tr><td>33.</td><td>Straight lines mean constant acceleration or deceleration</td></tr>
<tr><td>34.</td><td>Horizontal change means no change in velocity e.g. constant speed</td></tr>
<tr><td>35.</td><td>Positive gradient-= acceleration</td></tr>
<tr><td>36.</td><td>Negative gradient = deceleration</td></tr>
<tr><td>37.</td><td>Distance travelled = area under the graph</td></tr>
</table>

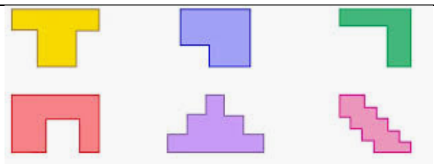
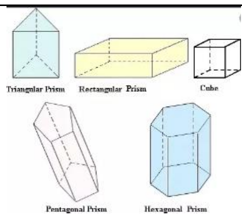
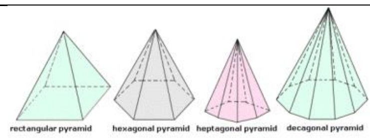
Quadratic, cubic and other graphs

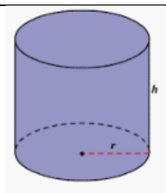
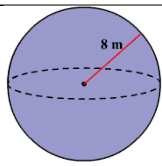
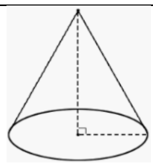
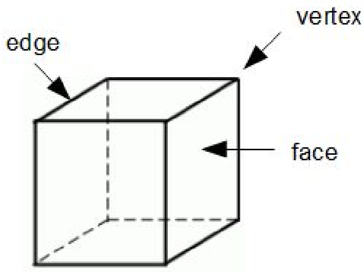
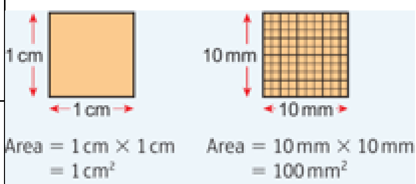
<table>
<tr><td>38.</td><td>Quadratic expression</td><td>An expression where the highest index is 2</td><td>e.g. $2x^2 + 2x + 2$</td></tr>
<tr><td rowspan="3">39.</td><td rowspan="3">Roots</td><td>Solutions to a quadratic equation/function
$ax^2 + bx + c = 0$</td><td rowspan="3">
</td></tr>
<tr><td>The x values where the graph crosses the x axis</td></tr>
<tr><td>A quadratic can have 0, 1 or 2 roots</td></tr>
<tr><td rowspan="3">40.</td><td rowspan="3">Quadratic graph</td><td>Curved shaped called a parabola</td><td rowspan="3">
</td></tr>
<tr><td>A positive x^2 will give a ‘∪’ shape</td></tr>
<tr><td>A negative x^2 will give a ‘∩’ shape</td></tr>
</table>

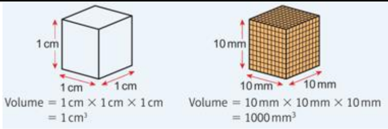
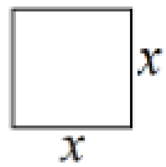
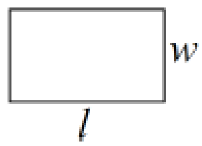
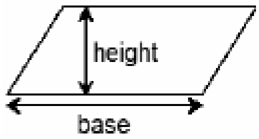
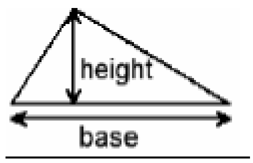
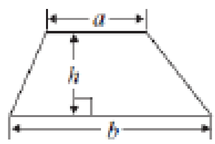
41.	Turning points	The point where a curve turns in the opposite direction Can be called a minimum or maximum	
42.	Cubic	General form of $ax^3 + bx^2 + cx + d = 0$ Can have 1, 2 or 3 roots	
43.	Asymptote	A line a graph will get very close to but will not touch	
44.	Reciprocal	General form of $y = \frac{k}{x}$ where k is a number Has two asymptotes	
45.	Circle	With centre $(0,0)$ and radius, r $x^2 + y^2 = r^2$	. $x^2 + y^2 = 16$ ($r = \sqrt{16} = 4$) 

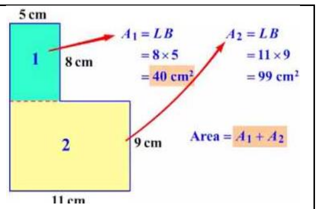
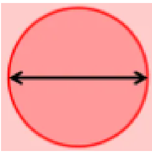
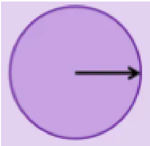
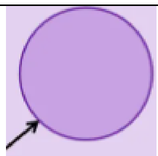
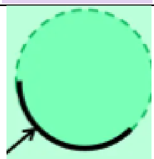
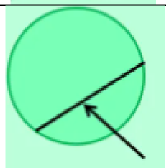
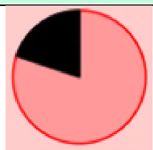
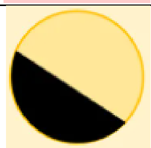
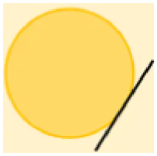
GCSE Mathematics
Higher
Unit 7

2D and 3D shapes: definitions

1.	Dimension	The size of something in a particular direction e.g. height, depth, length, width
2.	2D shape	A shape that has length/height and a width but no depth
3.	3D shape	A shape that depth as well as length/height and width
4.	Polygon	A 2D shape with straight lines only
5.	Regular polygon	A polygon where: All sides are the same length All angles are the same size
6.	Compound shape	A shape made up of two or more simple shapes
7.	Rectilinear shape	A shape where all of its sides meet at right angles 
8.	Perimeter	The distance around the outside of a 2D shape
9.	Area	The space inside a 2D shape
10.	Surface area	The total area of all the faces of a 3D shape
11.	Volume	The space inside a 3D shape
12.	Capacity	The amount of fluid a 3D object can hold
13.	S.I. Units	Standard units of measurement used by scientists across the world
14.	Metric units	Standard units of measurement that vary by powers of 10
15.	Imperial units	Older units of measurement, some of which are still common e.g. miles, gallons
16.	Cross section	The shape we get when cutting straight through a 3D shape
17.	Prism	A 3D shape that has a constant cross section through its length 
18.	Pyramid	A 3D shape with a polygon as its base and triangular sides that meet at the top 

<table>
<tr><td>19.</td><td>Cylinder</td><td colspan="4">A prism with two circular ends connected by a curved surface</td><td></td></tr>
<tr><td>20.</td><td>Sphere</td><td colspan="4">A 3D shape where all points on the surface are the same distance from the centre</td><td></td></tr>
<tr><td>21.</td><td>Spherical</td><td colspan="5">Means in the shape of a sphere</td></tr>
<tr><td>22.</td><td>Cone</td><td colspan="4">A 2D shape that has a circular base joined to a point by a curved side</td><td></td></tr>
<tr><td>23.</td><td>Face</td><td colspan="4">A flat surface of a 3D shape (can be curved)</td><td rowspan="3"></td></tr>
<tr><td>24.</td><td>Edge</td><td colspan="4">A line segment where two faces meet</td></tr>
<tr><td>25.</td><td>Vertex</td><td colspan="4">A point where two or more edges meet</td></tr>
<tr><td>26.</td><td>Vertices</td><td colspan="5">Plural of vertex</td></tr>
<tr><td colspan="7">Measures</td></tr>
<tr><td rowspan="2">27.</td><td rowspan="2">Units of time</td><td colspan="5">Standard units of time are seconds, minutes, hours, days, years</td></tr>
<tr><td>60 seconds = 1 minute</td><td>60 minutes = 1 hour</td><td>24 hours = 1 day</td><td colspan="2">365 days = 1 year</td></tr>
<tr><td rowspan="2">28.</td><td rowspan="2">Units of mass</td><td colspan="5">Metric units of mass are milligrams, grams, kilograms and tonnes</td></tr>
<tr><td>1000mg = 1g</td><td colspan="2">1000g = 1kg</td><td colspan="2">1000kg = 1 tonne</td></tr>
<tr><td rowspan="2">29.</td><td rowspan="2">Units of length</td><td colspan="5">Metric units of length are millimetres, centimetres, metres and kilometres</td></tr>
<tr><td>10mm = 1cm</td><td colspan="2">100cm = 1m</td><td colspan="2">1000m = 1km</td></tr>
<tr><td rowspan="3">30.</td><td rowspan="3">Units of area</td><td colspan="5">Metric units of length are millimetres^2, centimetres^2, metres^2 and kilometres^2</td></tr>
<tr><td colspan="4">$1cm^2 = 100mm^2$</td><td rowspan="2"></td></tr>
<tr><td colspan="4">$1m^2 = 1000cm^2$</td></tr>
</table>

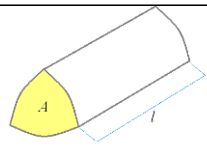
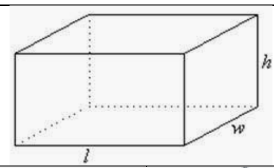
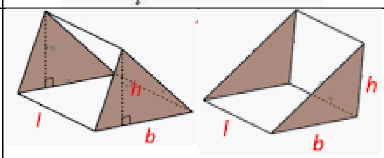
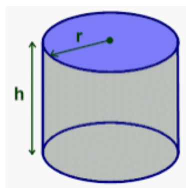
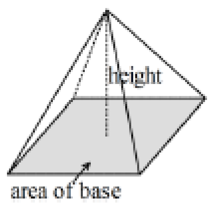
31.	Units of volume	Metric units of length are millimetres^3, centimetres^3, metres^3 and kilometres^3		
		$1cm^3 = 1000mm^3$		
		$1m^3 = 1000000cm^3$		
32.	Units of capacity	Metric units of capacity are millilitres, centilitres and litres		
		10*ml* = 1*cl*	1000*ml* = 100*cl* = 1*l*	
33.	Capacity and volume conversions	$1cm^3$ = 1*ml*	$1000cm^3$ = 1*l*	
2D Shapes				
34.	Square	Area = $l \times w$ or l^2 as length and width are equal		
35.		Perimeter = $l + l + l + l$ or $4l$		
36.	Rectangle	Area = $l \times w$		
37.		Perimeter = $l + l + w + w$ or $2l + 2w$		
38.	Parallelogram	Area = $b \times h$		
39.	Triangle	Area = $\frac{b \times h}{2}$ or $\frac{1}{2} \times b \times h$		
40.	Trapezium	Area = $\frac{a+b}{2} \times h$ or $\frac{1}{2}(a + b) \times h$		

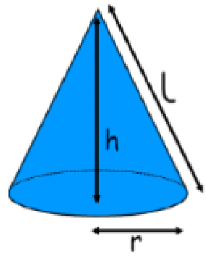
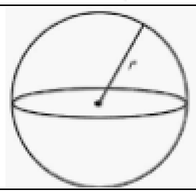
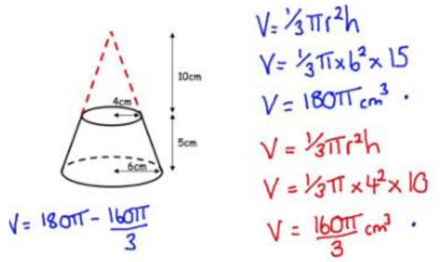
41.	Compound shape	To find the area, split up into simple shapes, find each area and add together. To find the perimeter, find any missing sides than add all the sides together.	
Circles			
42.	Diameter	A straight line from edge to edge passing through the centre Double the size of the radius	
43.	Radius	A straight line from the centre to the edge Half the size of the diameter	
44.	Radii	The plural of radius	
45.	Circumference	Distance around the outside of the circle	
46.	Arc	Part of the circumference	
47.	Chord	A line within a circle where each end touches the edge	
48.	Sector	The region created by two radii and an arc	
49.	Segment	The region created by a chord and an arc	
50.	Tangent	A line outside the circle which only touches the circumference at one point	
51.	Semi -circle	Half a full circle	

Area and circumference of circles formulae

52.	Pi (π)	Constant ratio linking the circumference and diameter of a circle 3.14159265…	
53.	Circumference of a circle	$C = \pi d$	Alternatively, using relationship between r and d $C = 2\pi r$
54.	Arc length	$\frac{x}{360} \times \pi d$	Where x is the angle at the centre
55.	Perimeter of a sector	$\left(\frac{x}{360} \times \pi d\right) + 2r$	This represents the arc length plus the two radii
56.	Area of a circle	$A = \pi r^2$	
57.	Area of a sector	$\frac{x}{360} \times \pi r^2$	

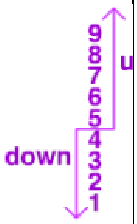
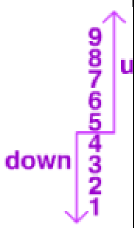
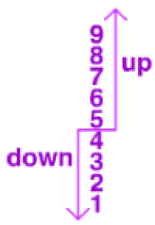
3D shapes: volume

58.	Prism	Volume = $area\ of\ cross\ section\ \times length$	
59.	Cuboid	Volume = $area\ of\ cross\ section\ \times length$ Volume = $length\ \times width\ \times height$	
60.	Triangular prism	Volume = $area\ of\ cross\ section\ \times length$ Volume = $\frac{1}{2} \times base\ \times height\ \times length$	
61.	Volume of a cylinder	$V = \pi r^2 h$	
62.	Surface area of a cylinder	$Total\ surface\ area = 2\pi r^2 + \pi dh$	
63.	Volume of a pyramid	$V = \frac{1}{3} \times area\ of\ base \times perpendicular\ height$	

64.	Volume of a cone	$V = \frac{1}{3} \times \pi r^2 h$	
65.	Surface area of a cone	$Curved\ surface\ area = \pi r l$ $Total\ surface\ area = \pi r^2 + \pi r l$	
66.	Volume of a sphere	$V = \frac{4}{3} \times \pi r^3$	
67.	Surface area of a sphere	$Total\ surface\ area = 4\pi r^2$	
68.	Volume of a frustum	Find the volume of the whole cones and subtract the volume of the smaller cone to get the volume of the frustum	

Accuracy and Bounds

69.	Integer	A whole number and the negative equivalents.
70.	Rounding	Changing a number to a simpler, easy to use value

71.	Round to a given number of decimal places	• Count the number of decimal places you need. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. 36.3486343 $36.3\|486343$ To 1 d.p. is 36.3 $36.34\|86343$ To 2 d.p. is 36.35 $36.348\|6343$ To 3 d.p. is 36.349
72	Round a large number to a given number of significant figures	• Count the number of digits you need from the left. • Look at the number to the right of that digit to decide if it rounds up or down. • 5 or more it rounds up, 4 or less it rounds down. • Replace remaining digits with zeros as place holders.		e.g. $324\,627\,938$ $3\|24627938$ To 1 s.f. is 300000000 $32\|4627938$ To 2 s.f. is 320000000 $324\|627938$ To 3 s.f. is 325000000
73.	Round a small number to a given number of significant figures	• Zeros are not significant until after the first non-zero number. • Find the first non-zero and count the number of digits you need from there. • Look at the number to the right of that digit to decide if it should round up or down. • 5 or more it rounds up, 4 or less it rounds down.		e.g. 0.0034792 $0.003\|4792$ To 1 s.f. is 0.003 $0.0034\|792$ To 2 s.f. is 0.0035 $0.00347\|92$ To 3 s.f. is 0.00348

74.	Estimating	• Round each number to 1 significant figure before doing any calculations. • It is acceptable to round one or more numbers in the calculation to a greater accuracy than 1 sig. fig. if this makes the calculation easier. • DO NOT round the answer!	e.g. Estimate: $\frac{\mathbf{3.91 \times 8789.8}}{\mathbf{620.9 \times 0.492}}$ $\frac{3.91 \times 8789.8}{620.9 \times 0.492} \approx \frac{4 \times 9000}{600 \times 0.5}$ $\approx \frac{3600}{300}$ $\approx \mathbf{120}$
75.	Truncation	Approximating a number by ignoring all decimal points after a certain point without rounding	e.g. 5.6 would be 5 when truncated
76.	Error interval	Measurements measured to the nearest unit may be up to half a unit smaller or larger than the rounded value	e.g. If 5.6 is rounded correct to the nearest 1dp then the interval is $5.55 \leq x < 5.65$
77.	Upper bound	The upper bound is half a unit greater than the rounded number	e.g. the upper bound of 5.6 when measured to the nearest 1dp is 5.65
78.	Lower Bound	The lower bound is half a unit less than the rounded number	e.g. the lower bound of 5.6 when measured to the nearest 1dp is 5.55
79.	Appropriate accuracy	The accuracy when both the upper and lower bound are rounded by the same amount and give the same value e.g. If UB = 12.3512 and LB = 12.3475 To 1dp: UB = 12.4 and LB- 12.3 To 2dp: UB = 12.35 and LB – 12.35 To 3dp: UB = 12.351 and LB =12.348	Here the appropriate accuracy is 2 dp

GCSE Mathematics
Higher
Unit 8

Transformations - definitions

1.	Transformation	Changing a 2D shape in some way.			
		Rotation	Reflection	Translation	Enlargement
2.	Object	The name given to a shape before a transformation has occurred.			
3.	Image	The name given to a shape after a transformation has occurred			
4.	Rotation	A circular movement about a fixed point			
5.	Centre of rotation	The fixed point that the shape has been rotated about			
		Written as a coordinate (x, y)			
6.	Direction	Clockwise or anticlockwise			
7.	Reflection	An image as it would be seen in a mirror			
8.	Line of reflection	The "mirror line" used to perform reflections.			
		Written using algebraic notation e.g. $y = 3$, $x = -2$, $y = x$ or x/y axis			
9.	Translation	The movement of a shape without rotating or flipping it			
10.	Column vector	Notation used to represent translations		$\left(\frac{x}{y}\right)$	
		x is the horizontal movement			
		y is the vertical movement			
11.	Resultant vector	The vector that moves the shape to its final position after more than one translation			
12.	Enlargement	A change in size of a shape (can be bigger or smaller)			
13.	Scale factor	The proportions by which the dimensions of an object will increase/decrease by			
		If fractional then the image will be smaller than the object			
14.	Negative scale factor	The image will be on the opposite side of the centre of enlargement			
15.	Centre of enlargement	A fixed point to enlarge an object from			
		Written as a coordinate (x, y)			
16.	Single transformation	Where the object is only transformed once			
17.	Combination	Where the object is transformed multiple times			
18.	Origin	The point (0,0); where the x and y axis intersect			
19.	Similar	Same shape but different sizes			

		e.g. similar shapes are enlargements of one another
20.	Congruent	Shapes that are the same shape and size
21.	Invariant	A property that does not change after a transformation
22.	Invariant point	A point that does not change after a transformation
23.	Describe	Use key words to accurately state what has happened to an object to make the resulting image

Transformations

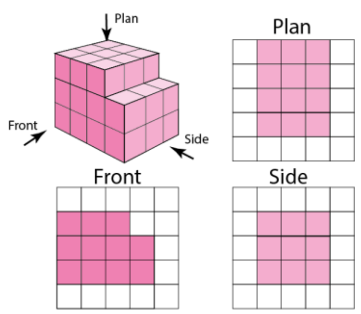
	Rotation	To carry out you need to: 1. Draw object on tracing paper 2. Place pencil on 'centre of rotation' and carry out the motion 3. Draw your image on the grid	To describe you need to write: a) "rotation" b) angle of rotation c) direction of rotation d) centre of rotation
	Reflection	To carry out you need to: 1. If required draw the 'line of reflection' 2. Count squares from object to line and repeat the other side of the line for all corners of the object 3. Join points up to create the image	To describe you need to write: a) "reflection" b) the equation of the line of reflection
	Translation	To carry out you need to: 1. Use vector notation to work out the horizontal and vertical movement 2. Count squares to carry out movement on all corners of the object 3. Join up points to create the image	To describe you need to write: a) "translation" b) the column vector
	Enlargement	To carry out you need to: 1. If required cross the coordinate that is the centre of enlargement 2. For each corner count from the line of reflection to the object 3. Multiply this movement by the required scale factor 4. Draw new corners from the centre of enlargement with new	To describe you need to write: a) "enlargement" b) the scale factor c) the centre of enlargement

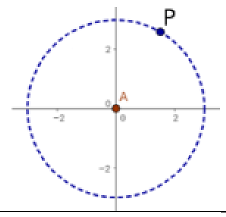
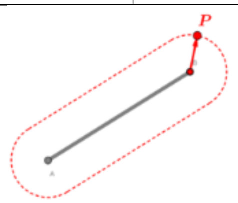
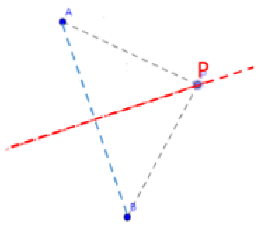
		horizontal and vertical movement 5. Join up points to create image	

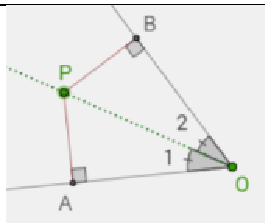
2D shapes and 3D solids - definitions

1.	Face	A flat surface of a 3D shape
2.	Edge	A line segment where two faces meet
3.	Vertex	A point where two or more edges meet
4.	Vertices	The plural of vertex
5.	Dimension	The size of something in a particular directions e.g. length, width, height, diameter, depth
6.	Plane	A flat 2D surface
7.	Plane of symmetry	When a solid can be cut exactly in half and a part on one side of the plane is an exact reflection of the part on the other side of the plane
8.	Prism	A 3D shape with a uniform cross section
9.	Pyramid	A 3D shape with a polygon as a base and triangular sides that meet at the top
10.	Arc	A section from the circumference (outside) of a circle
11.	Sector	A region of a circle bound by two radii and an arc
12.	Congruent	Exactly the same shape and size e.g. identical
13.	Regular	A shape where all the sides and angles are the same

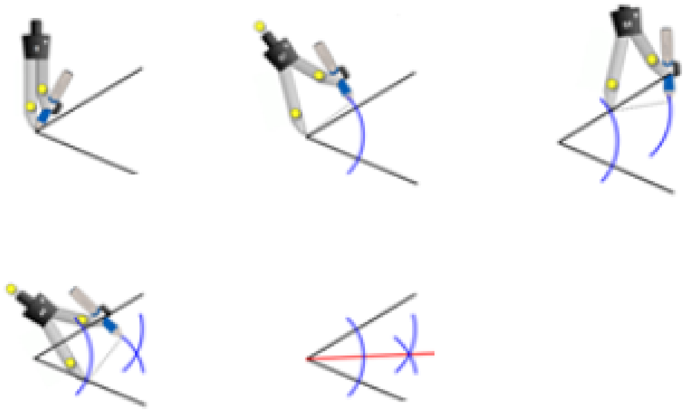
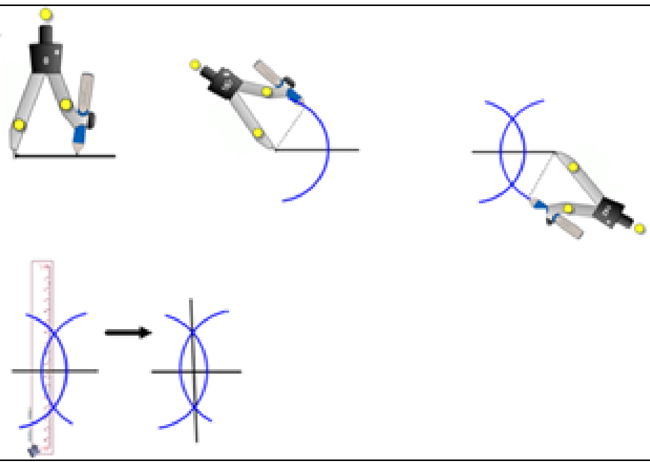
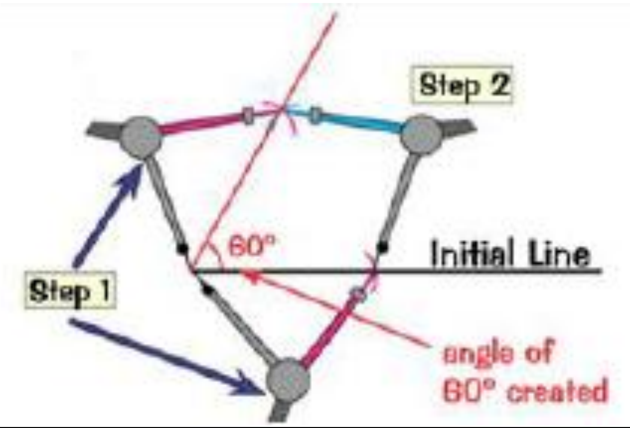
Plans and elevations

14.	Plan	The view from above a solid	
15.	Front elevation	The view from the front of a solid	
16.	Side elevation	The view from a side of the solid	
17.	Clockwise	Following the direction of a clock	
18.	Anticlockwise	Following the opposite direction of a clock	
19.	Compass directions	Terminology needed to accurately describe a location or directions	

20.	Sketch	An approximate drawing of an object	
21.	Scale	A ratio that shows the relationship between a length on a drawing/map and the actual length	
Constructions and loci			
22.	Construct	Draw accurately using a ruler and a pair of compasses.	
23.	Construction lines	Lines or arcs drawn as part of working out	
		They must not be rubbed out as they show the working	
24.	Equidistant	The same distance from each other or in relation to other things	
25.	Bisect	Cut in half	
26.	Perpendicular	At a 90 degree angle (right angle)	
27.	Perpendicular bisector	A line that cuts another in half at a right angle	
28.	Angle bisector	A line that cuts an angle exactly in half	
29.	Locus	The set of all points that fulfil a certain rule	
		Often drawn as a continuous path	
30.	Loci	The plural of locus	
31.	Region	An area bounded by a loci	
Loci			
32.	Circle	Locus of points that are a fixed distance from a fixed point	
33.	Parallel line	Locus of points a fixed distance from a fixed line	
34.	Perpendicular bisector	The line that cuts another in half at a right angle	

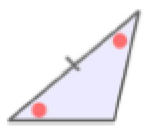
35.	Angle bisector	The locus of points equidistant between two fixed points.	

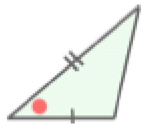
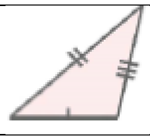
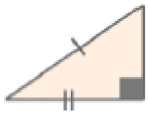
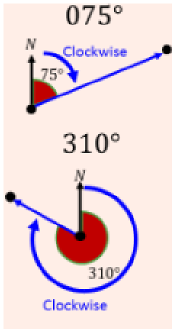
Constructions

36.	Angle bisector	
37.	Perpendicular bisector	
38.	Constructing 60° angles	

Constructing triangles

You can draw an accurate triangle when you are given:

39.	ASA	an angle, side, angle	

40.	SAS	a side, angle, side	
41.	SSS	all three sides	
42.	RHS	that it has a right angle, the hypotenuse and another side	
Bearings			
43.	Bearing	The direction of a line in relation to the North-South line It is always measured clockwise Always measured from the North line Always written using 3 digits	

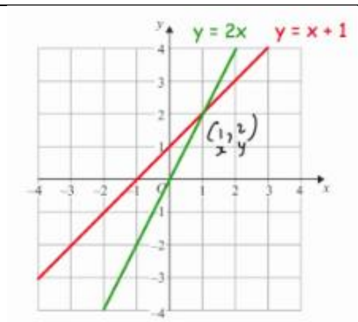
GCSE Mathematics Higher Unit 9

Factorising a quadratic expression

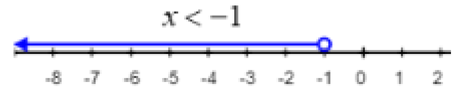
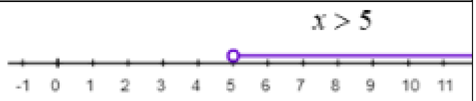
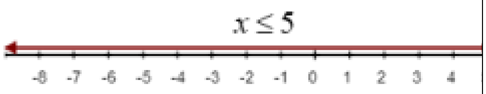
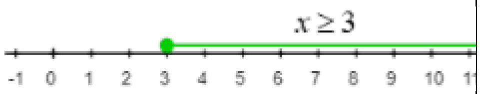
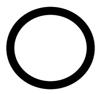
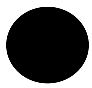
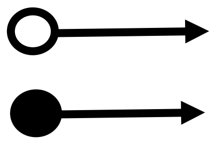
1.	Factorising a quadratic in the form of $ax^2 + bx + c$	Multiply to 5 / Factorise $x^2 + 5x + 6$ ← Add to 6 2 and 3 add to 5 2 and 3 multiply to 6 $(x+2)(x+3)$ Check: $(x+2)(x+3) = x^2 + 5x + 6$	
2.	Difference of two squares	A special type of quadratic which only has two terms.	
		One term is subtracted from the other	
		$x^2 - 25 = x^2 - 5^2 = (x+5)(x-5)$ $y^2 - 49 = y^2 - 7^2 = (y+7)(y-7)$ $a^2 - 16 = a^2 - 4^2 = (a+4)(a-4)$	
3.	Factorising a quadratic in the form of $ax^2 + bx + c$ where $a > 1$	By inspection $4x^2 + 20x + 9$ $(4x+9)(x+1)$ $(4x+3)(x+3)$ $(2x+9)(2x+1)$ ✓ $(2x+3)(2x+3)$	Splitting the middle $4x^2 + 20x + 9$ $4x^2 + 2x + 18x + 9$ $\mathbf{2x}(2x+1) + \mathbf{9}(2x+1)$ $(2x+1)(2x+9)$

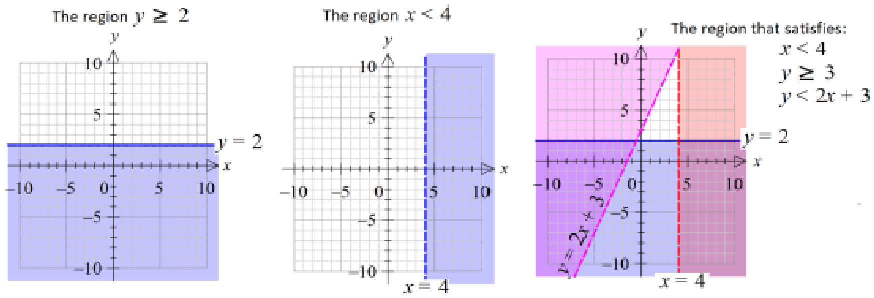
Solving quadratic equations/functions

4.	By factorising	Take you factorised form and set each bracket equal to zero	$x^2 + 4x + 3 = 0$ $(x+3)(x+1) = 0$ $x + 3 = 0$ So $x = -3$ $x + 1 = 0$ So $x = -1$
		Solve each separate linear equation to find the solutions/roots	
5.	The quadratic formula	A formula to find the solutions a quadratic equation in the form of $ax^2 + bx + c$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

<table>
<tr><td>6.</td><td>Completing the square</td><td colspan="2">$x^2 + bx + c$ can be written in the form
$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$</td><td>If a is greater than 1 this will need to be factored out first!</td></tr>
<tr><td colspan="5"><h2>Simultaneous equations</h2></td></tr>
<tr><td>7.</td><td>Simultaneous equations</td><td colspan="3">Two equations where there are two unknown which have the same value in each</td></tr>
<tr><td colspan="5"><h3>Solving simultaneous equations</h3></td></tr>
<tr><td rowspan="2">8.</td><td rowspan="2">Elimination</td><td colspan="3">Add or subtract one equation from another to eliminate a variable</td></tr>
<tr><td colspan="2">If the matching coeefieicents have the same sign then subtract the equations
✓ Same
✓ Subtract
✓ Substitute</td><td>If the matching coefficients have different signs then add the equations
✓ Different
✓ Add
✓ Substitute</td></tr>
<tr><td rowspan="2">9.</td><td rowspan="2">Substitution</td><td colspan="3">Rearrange so the subject of one equation is a single variable</td></tr>
<tr><td colspan="3">Substitute this into the second equation</td></tr>
<tr><td>10.</td><td>Graphically</td><td colspan="2">The points of intersection of two graphs are the solutions to the simultaneous equations</td><td></td></tr>
</table>

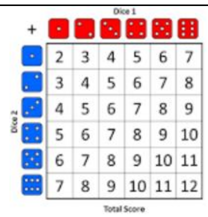
Inequalities

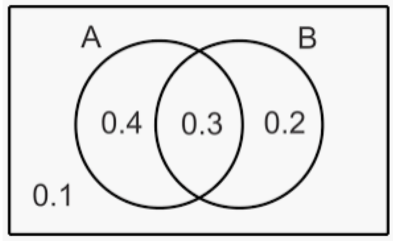
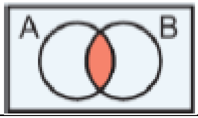
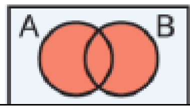
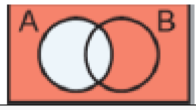
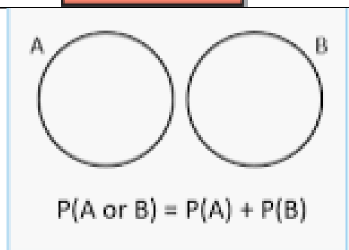
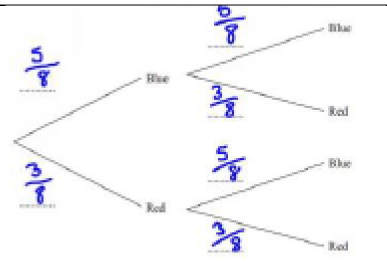
11.	Inequality	The relationship between two expressions that are not equal	
12.	=	Equal to	
13.	≠	Not equal to	
14.	<	Less than	
15.	>	Greater than	
16.	≤	Less than or equal to	
17.	≥	Greater than or equal to	
18.	Inclusive	Gives a finites rnage of solutions	e.g. $3 < x \leq 8$
19.	Exclusive	Gives an infinite range of solutions	e.g. $x > 5$ $-4 \leq x$
20.	Integer	A whole number that can be positive negative or zero	
21.	Solve	Inequalities are solved in the same way as solving equations	
		Only exception: if you multiply or divide by a negative number you must swap the sign e.g. less than to greater than	
22.	List integers solutions	Give the integers that satisfy the inequality	
		e.g. x > 6 integer solutions are 6, 7, 8....	
		e.g. -5 < x ≤ 5 integer solutions are -4, -3, -2, -1, 0, 1, 2, 3, 4, 5	
23.	Represent on a number line	An empty circle shows the value is not included	
		A shaded circle shows the value is included	
		An arrow shows that the solution continues to infinity	

24.	Inequalities on graphs	 The unwanted sections are shaded
		Dashed lines are used to represent < or >
		Solid line is used to represent $\leq$ or $\geq$

GCSE Mathematics
Higher
Unit 10

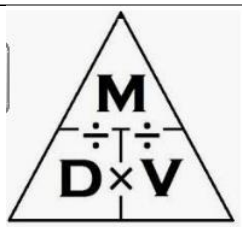
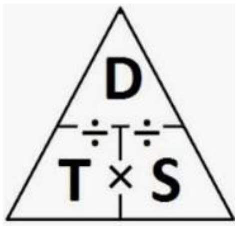
Probability - definitions

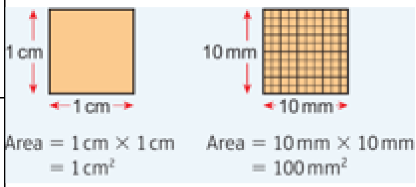
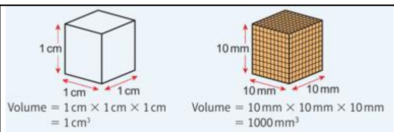
1.	Probability	The extent to which an event is likely to occur Written as a fraction, decimal or percentage	For equally likely outcomes the probability that an event will happen is $P = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ possible\ outcomes}$
2.	Theoretical probability	Calculated without doing an experiment	
3.	Experimental probability	Probabilities based on the data collected during an experiment Also known as estimated probability The more trials you do the more reliable your set of results	$estimated\ probability = \frac{frequency\ of\ event}{total\ frequency}$
4.	P( ) notation	P( ) mean s the probability of the thing inside the brackets happening e.g. P(tails)	
5.	Experiment	A repeatable process that gives rise to a number of outcomes	
6.	Relative frequency	In an experiment, how often something happens as a proportion of the number of trials	Relative frequency = $\frac{how\ often\ something\ happens}{all\ outcomes}$
7.	Predictions	You can predict the number of outcomes you will get using relative frequency Predicted number of outcomes = probability x number of trials	
8.	Event	A collection of one or more outcomes	
9.	Independent	When one event has no effect on another	Here P(A and B) = P(A) x P(B)
10.	Dependent	When the outcome of one event, changes the probability of the next event	
11.	Exhaustive	Events are exhaustive if they cover all possible outcomes	
12.	Biased	Unfair	
13.	Unbiased	Fair	
14.	Sample space	The set of all possible outcomes	
15.	Sample space diagram	A diagram showing all possible outcomes from an experiment	

16.	Venn diagram	Can be used to represent events graphically Frequencies or probabilities can be placed in the regions		
17.	A ∩ B	A intersection B	All elements in A and B	
18.	A ∪ B	A union B	All the elements in A OR B OR both	
19.	A'	Complement of A	Not in A	
20.	Mutually exclusive	Events that have no outcomes in common Here P(A or B) = P(A) + P(B)		
21.	Tree diagram	Used to show the outcomes of two (or more) events happening in succession		
22.	AND rule	Multiply the probabilities		
23.	OR rule	Add the probabilities		
24.	Conditional probability	The probability of a dependent event The probability of a second outcome depends on what has already happened in the first outcome		

GCSE Mathematics Higher Unit 11

Multiplicative reasoning – definitions and formulae

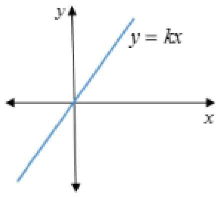
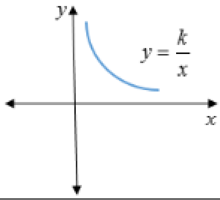
1.	Proportion	Compares a part with a whole	
2.	Proportional	A change in one is always accompanied by a change in another	
3.	Ratio	A relationship between two or more quantities	
4.	Compound measure	Combine measures of two different quantities	
5.	Density	The mass of a substance contained in a certain volume Usually measured in g/cm^3 or kg/m^3 $density = \frac{mass}{volume}$	
6.	Velocity	Speed in a given direction	Usually measured in m/s
7.	Acceleration	The rate of change of velocity	Usually measured in m/s^2
8.	Speed	The distance travelled in an amount of time Usually measured in m/s, mph or km/h $speed = \frac{distance}{time}$	
9.	Pressure	The force applied over an area $pressure = \frac{force}{area}$ Usually measured in N/m^2	
10.	Units of time	Standard units of time are seconds, minutes, hours, days, years 60 seconds = 1 minute; 60 minutes = 1 hour; 24 hours = 1 day; 365 days = 1 year	
11.	Units of mass	Metric units of mass are milligrams, grams, kilograms and tonnes 1000mg = 1g; 1000g = 1kg; 1000kg = 1 tonne	

12.	Units of length	Metric units of length are millimetres, centimetres, metres and kilometres		
		10mm = 1cm	100cm = 1m	1000m = 1km
13.	Units of area	Metric units of length are millimetres^2, centimetres^2, metres^2 and kilometres^2		
		1cm^2 = 100mm^2		
		1m^2 = 1000cm^2		
14.	Units of volume	Metric units of length are millimetres^3, centimetres^3, metres^3 and kilometres^3		
		1cm^3 = 1000mm^3		
		1m^3 = 1000000cm^3		
15.	Units of capacity	Metric units of capacity are millilitres, centilitres and litres		
		10*ml* = 1*cl*	1000*ml* = 100*cl* = 1*l*	
16.	Capacity and volume conversions	1cm^3 = 1*ml*	1000cm^3 = 1*l*	

Percentages

17.	Percentage	Means 'out of 100'	
18.	Multiplier	A decimal you multiply by to represent a percentage	
		To use a multiplier to find a percentage, divide your percentage by 100, then multiply the amount by this value.	
19.	Percentage increase	Calculate the percentage and add onto the original	
		Or use a multiplier	$amount \times \frac{100 + \%\ increase}{100}$
20.	Percentage decrease	Calculate the percentage and subtract from the original	
		Or use a multiplier	$amount \times \frac{100 - \ \%\ increase}{100}$
21.	Percentage change	$\frac{Change}{Original} \times 100$	
22.	Express one number as a percentage of another	$\frac{Number\ 1}{Number\ 2} \times 100$	

23.	Reverse percentage	Use when asked to find the priginal amount after a percentage increase or decrease. Original Value x Multiplier = New Value $\text{Original Value} = \dfrac{\text{New Value}}{\text{Multiplier}}$	
24.	Interest	A fee paid for borrowing money or money earnt through investing.	
25.	Simple interest	Interest that is calculated as a percentage of the original	I = Prt I – Interest P – Original amount r – interest rate t - time
26.	Compound interest	When interest is calculate on the original amount and any previous interest Or $Original \times Multiplier^{time}$	$P\left(1+\frac{R}{100}\right)^n$ P – Original amount R – Interest rate n – the number of interest periods (e.g. yrs)
27.	Tax	A financial charge placed on sales or savings by the government e.g. VAT	
28.	Loss	Income minus all expenses, resulting in a negative value	
29.	Profit	Income minus all expenses, resulting in a positive value	
30.	Depreciation	A reduction in the value of a product over time	
31.	Annual	Means yearly	
32.	Per annum	Means per year	
33.	Salary	A fixed regular payment, often paid monthly	

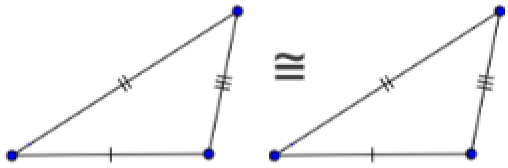
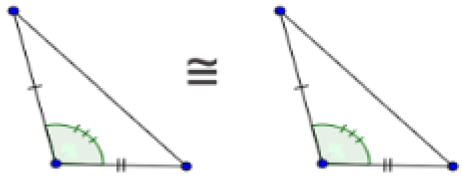
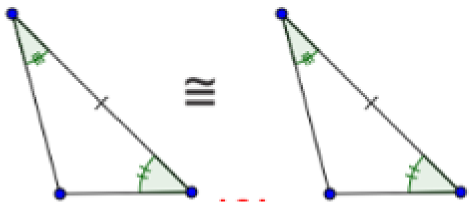
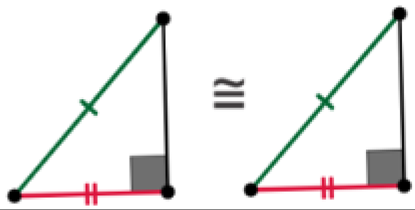
<table>
<tr><th colspan="4">Proportion graphs</th></tr>
<tr><td rowspan="2">34.</td><td rowspan="2">Direct proportion</td><td>Two quantities increase at the same rate</td><td rowspan="2">$y \propto x$
$y = kx$ for a constant k

</td></tr>
<tr><td>Graph is a straight line that goes through the origin</td></tr>
<tr><td>35.</td><td>Inverse/indirect proportion</td><td>One variable increases at a constant rate as the second variable decreases</td><td>$y \propto \frac{1}{x}$
$y = \frac{k}{x}$ for a constant k

</td></tr>
<tr><td rowspan="2">36.</td><td rowspan="2">Constant of proportionality</td><td colspan="2">Represented by k</td></tr>
<tr><td colspan="2">Its value stays the same</td></tr>
</table>

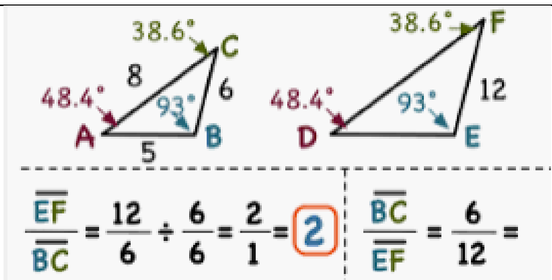
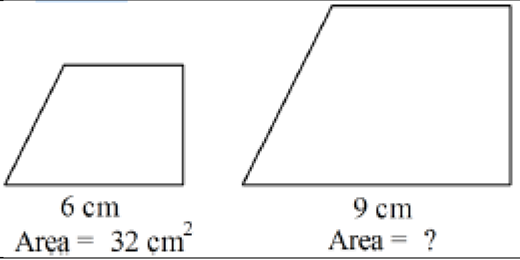
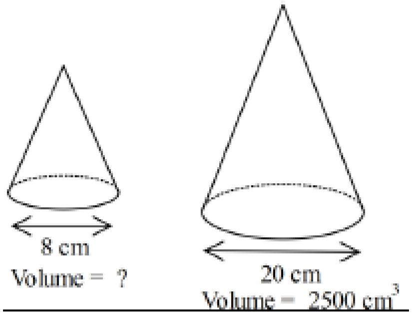
GCSE Mathematics
Higher
Unit 12

Similarity and Congruence in 2D and 3D

1.	Congruent	Exactly the same shape and size	
2.	Similar	Two shapes where one is an enlargement of another	
		Corresponding angles are equal	Corresponding sides are in the same ratio
3.	Scale factor	The proportion by which the dimensions of an object will increase or decrease by	
4.	Linear scale factor (LSF)	The scale factor/ratio of sides of two similar shapes	$LSF = \frac{length\ from\ large\ shape}{length\ from\ small\ shape}$
5.	Area scale factor (ASF)	The scale factor ratio of areas/surface areas of two similar shapes	$ASF = \frac{Area\ of\ large\ shape}{lArea\ of\ small\ shape}$
6.	Volume scale factor (VSF)	The scale factor/ratio of volumes of two similar shapes	$VSF = \frac{volume\ of\ large\ shape}{volume\ of\ small\ shape}$

Two triangles are congruent if…

7.	SSS	All 3 sides are equal	
8.	SAS	2 sides and the included angle are equal	
9.	ASA	2 angles and the corresponding side are equal	
10.	RHS	The right angle, hypotenuse and one other side are equal	

Similar shapes			
11.	Lengths		The scale factor from small to big is 2.
12.	Areas		LSF = 9÷6 =1.5 ASF = 1.5^2 So area of bigger shapes is 6 x 1.5^2
13.	Volumes		LSF = 20 ÷8 = 2.5 VSF = 2.5^2 So volume of smaller shape is 2500 ÷ 2.5^2

GCSE Mathematics Higher Unit 13

Graph transformations

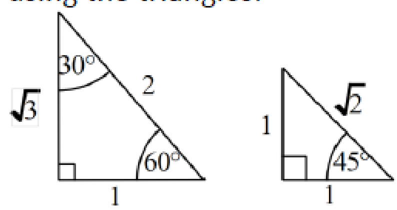
1.	$y = -f(x)$	Reflection in the x axis	y coordinates are multiplied by -1
2.	$y = f(-x)$	Reflection in the y axis	x coordinates are divided by -1
3.	$y = -f(-x)$	Reflection in the x axis and then in the y axis Equivalent to rotation of 180◦ about the origin	y coordinates are multiplied by -1 AND x coordinates are divided by -1
4.	$y = f(x) + a$	Translation by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$	
5.	$y = f(x + a)$	Translation by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$	
6.	$y = af(x)$	Stretch by scale factor a in the vertical direction, parallel to the y axis	y coordinates are multiplied by a
7.	$y = f(ax)$	Stretch by scale factor $\frac{1}{a}$ in the horizontal direction, parallel to the x axis	x coordinates are multied by $\frac{1}{a}$

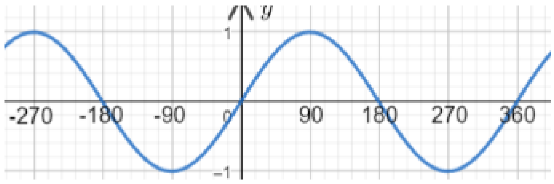
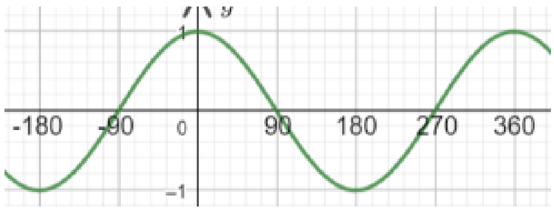
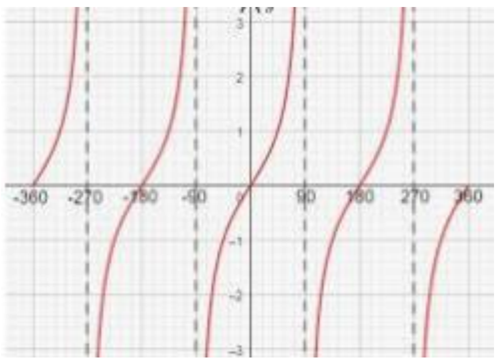
Exact Trig values

8. Exact Values

ϴ	0°	30°	45°	60°	90°
Sin ϴ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos ϴ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan ϴ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

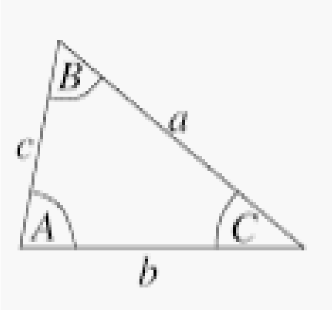
These can be found using the triangles:



<table>
<tr><td colspan="4">Trigonometric graphs</td></tr>
<tr><td rowspan="3">9.</td><td rowspan="3">Sine graph</td><td>Repeats every 360°</td><td rowspan="3">
</td></tr>
<tr><td>Crosses the x-axis at -180°, 0°, 180°, 360°...</td></tr>
<tr><td>Maximum of 1 and minimum of -1</td></tr>
<tr><td rowspan="3">10.</td><td rowspan="3">Cosine graph</td><td>Repeats every 360°</td><td rowspan="3">
</td></tr>
<tr><td>Crosses x-axis at -90°, 90°, 270°, 450°...</td></tr>
<tr><td>Maximum of 1 and minimum of -1</td></tr>
<tr><td rowspan="4">11.</td><td rowspan="4">Tangent graph</td><td>Repeats every 180°</td><td rowspan="4">
</td></tr>
<tr><td>Crosses x-axis at -180°, 0°, 180°, 360°...</td></tr>
<tr><td>Has no maximum or minimum value</td></tr>
<tr><td>Has vertical asymptotes at x=-90°, x=90°, x=270°...</td></tr>
<tr><td colspan="4">Non – right angled trigonometry</td></tr>
<tr><td>12.</td><td>Cosine rule</td><td>Finding sides
$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$</td><td>Finding angles
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$\cos C = \frac{a^2 + b^2 - c}{2ab}$</td></tr>
<tr><td>13.</td><td>Sine rule</td><td colspan="2">Finding sides
$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Finding angles
$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Ambiguous case
Can sometimes produce two possible solutions for missing angles
$\sin\theta = \sin(180 - \theta)$</td></tr>
</table>

14.	**Area of a triangle**	$Area = \frac{1}{2}ab\sin C$ $Area = \frac{1}{2}bc\sin A$ $Area = \frac{1}{2}ac\sin B$	

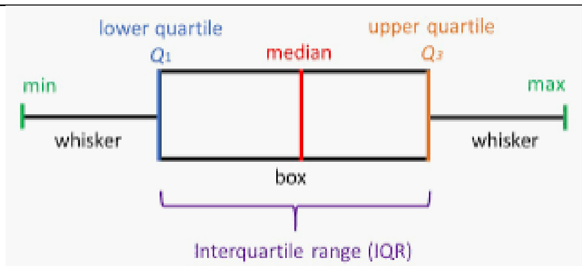
Collecting data

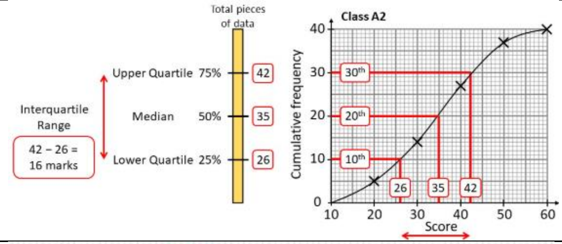
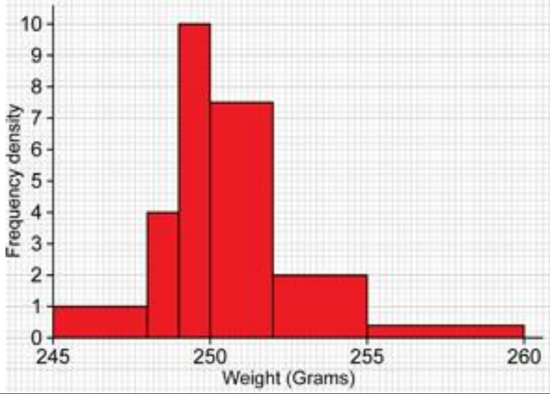
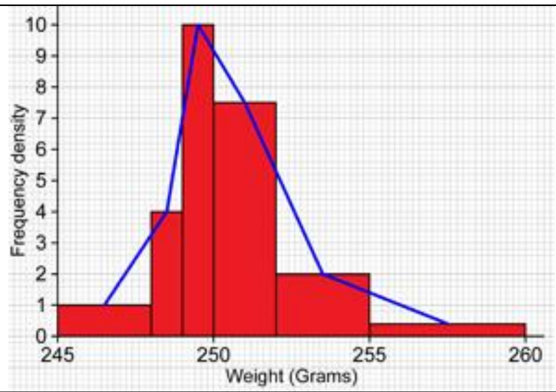
<table>
<tr><td>1.</td><td>Population</td><td colspan="2">The whole set of items that are of interest e.g. all the people in a school</td></tr>
<tr><td rowspan="2">2.</td><td rowspan="2">Census</td><td colspan="2">Observes or measures every member of a population.</td></tr>
<tr><td>Advantages
• Should give a completely accurate result</td><td>Disadvantages
• Time consuming
• Hard to process such large quantities of data
• Cannot be sued when the testing process destroys the item</td></tr>
<tr><td rowspan="2">3.</td><td rowspan="2">Sample</td><td colspan="2">A collection of observations taken from the subset of the population which is then used to find out information of the population as a whole</td></tr>
<tr><td>Advantages
• Less time consuming and expensive than a census
• Fewer people have to respond
• Less data to process compared to a census</td><td>Disadvantages
• Data may not be as accurate
• Sample may not be large enough to give information about smaller sub groups in the population</td></tr>
<tr><td>4.</td><td>Sampling units</td><td colspan="2">Individual units of a population</td></tr>
<tr><td>5.</td><td>Sampling frame</td><td colspan="2">The list of people or items to be sampled</td></tr>
<tr><td>6.</td><td>Stratum</td><td colspan="2">A subset of the population which is being sampled</td></tr>
<tr><td>7.</td><td>Strata</td><td colspan="2">Plural of stratum</td></tr>
<tr><td>8.</td><td>Bias</td><td colspan="2">Prejudice for or against one group or opinion or result in a way that is unfair</td></tr>
</table>

Random sampling techniques

<table>
<tr><td rowspan="2">9.</td><td rowspan="2">Simple random sampling</td><td colspan="2">Where every member of the sampling frame has an equal chance of being selected.</td></tr>
<tr><td>Advantages
• Free of bias
• Easy and cheap to implement for small populations and samples</td><td>Disadvantages
• Not suitable when population size or sample size is large
• A sampling frame is needed</td></tr>
</table>

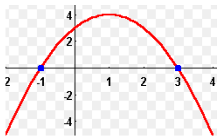
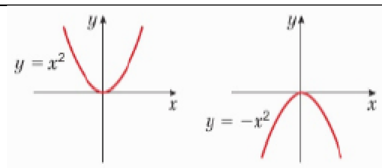
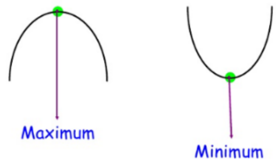
<table>
<tr><td rowspan="2">10.</td><td rowspan="2">Systematic sampling</td><td colspan="2">Where required elements are chosen at regular intervals from an ordered list</td></tr>
<tr><td>Advantages
Simple and quick to use
Suitable for large samples and populations</td><td>Disadvantages
A sampling frame is needed
It can introduce bias if the sampling frame is not random</td></tr>
<tr><td rowspan="3">11.</td><td rowspan="3">Stratified sampling</td><td colspan="2">The population is divided into mutually exclusive strata (e.g. males and females) and a random sample is taken from each</td></tr>
<tr><td colspan="2">$$Number\ sample\ in\ a\ stratum = \frac{number\ in\ stratum}{number\ in\ population} \times overall\ sample\ size$$</td></tr>
<tr><td>Advantages
Sample accurately reflects the population structure
Guarantees proportional representation of groups within a population</td><td>Disadvantages
Population must be clearly classified into distinct strata
Selection within each stratum suffers from the same disadvantages as simple random sampling</td></tr>
<tr><td colspan="4"><h2>Non- random sampling techniques</h2></td></tr>
<tr><td rowspan="2">12.</td><td rowspan="2">Quota sampling</td><td colspan="2">A researcher selects a sample that reflects the characteristics of the whole population</td></tr>
<tr><td>Advantages
Allows a small sample to be representative of the whole population
No sampling frame required
Quick, easy and inexpensive
Allows for easy comparison between different groups in a population</td><td>Disadvantages
Non random sampling can introduce bias
Population must be divided into groups which can be costly or inaccurate
Increasing scope of study increases number of groups, which adds time and expense
Non-responses are not recorded as such</td></tr>
<tr><td rowspan="3">13.</td><td rowspan="3">Opportunity sampling</td><td colspan="2">Taking the sample from people who are available at the time the study is carried out and who fit the criteria you are looking for</td></tr>
<tr><td colspan="2">Also known as ‘convenience sampling’</td></tr>
<tr><td>Advantages
Easy to carry out
Inexpensive</td><td>Disadvantages
Unlikely to provide a representative sample
Highly dependent of the individual researcher</td></tr>
</table>

<table>
<tr><th colspan="3">Types of data</th></tr>
<tr><td>14.</td><td>Quantitative data (or variables)</td><td>Data (or variables) associated with numerical observations e.g. shoe size</td></tr>
<tr><td>15.</td><td>Qualitative date (or variables)</td><td>Data (or variables) associated with non-numerical observations e.g. hair colour</td></tr>
<tr><td>16.</td><td>Continuous variable (data)</td><td>A variable that can take any value in a given range e.g. time</td></tr>
<tr><td>17.</td><td>Discrete variable (data)</td><td>A variable that can take only specific values in a given range e.g. number of girls in a family</td></tr>
<tr><th colspan="3">Representing and interpreting data</th></tr>
<tr><td>18.</td><td>Class</td><td>Another name for the groups in a grouped frequency table</td></tr>
<tr><td>19.</td><td>Class boundaries</td><td>The maximum and minimum values that belong in each class</td></tr>
<tr><td>20.</td><td>Class width</td><td>The difference between the upper and lower class boundaries</td></tr>
<tr><td>21.</td><td>Midpoint</td><td>The average of the class boundaries</td></tr>
<tr><td>22.</td><td>Outlier</td><td>An extreme value that lies outside the overall pattern of the data</td></tr>
<tr><td>23.</td><td>Anomalies</td><td>Any outliers that should be removed from the data because it is an error and it would be misleading to keep it in</td></tr>
<tr><th colspan="3">Types of graphs/charts</th></tr>
<tr><td>24.</td><td>Box plots</td><td>A diagram that displays median, quartiles, minimum and maximum values of a set of data

</td></tr>
<tr><td>25.</td><td>Cumulative frequency</td><td>A running total of frequencies</td></tr>
<tr><td>26.</td><td>Cumulative frequency table</td><td>A table that shows how many data items are less than or equal to the upper class boundary of each data class
<table>
<tr><th>Time, t (minutes)</th><th>Frequency</th><th>Cumulative Frequency</th></tr>
<tr><td>$0 < t \leq 20$</td><td>16</td><td>16</td></tr>
<tr><td>$20 < t \leq 30$</td><td>24</td><td>40</td></tr>
<tr><td>$30 < t \leq 50$</td><td>19</td><td>59</td></tr>
<tr><td>$50 < t \leq 80$</td><td>8</td><td>67</td></tr>
</table></td></tr>
</table>

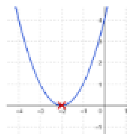
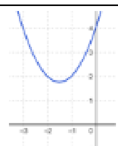
<table>
<tr><td>27.</td><td>Upper class boundary</td><td colspan="2">The highest possible value in each class</td></tr>
<tr><td>28.</td><td>Cumulative frequency graph</td><td>A graph with the data values on the x axis and the cumulative frequency on the y axis</td><td>
</td></tr>
<tr><td rowspan="2">29.</td><td rowspan="2">Histogram</td><td>A chart where the area of each bar is proportional to the frequency of each class</td><td rowspan="2">
</td></tr>
<tr><td>Area of each bar = k x frequency (k = 1 is the easiest value to use when drawing a histogram)</td></tr>
<tr><td>31.</td><td>Frequency density</td><td>The height of each bar on a histogram</td><td>If k = 1 then:
$$frequency\ density = \frac{frequency}{class\ width}$$</td></tr>
<tr><td>31.</td><td>Frequency polygon</td><td>Can be formed by joining the middle of each bar in a histogram</td><td>
</td></tr>
</table>

GCSE Mathematics Higher Unit 15

Quadratics - definitions

2.	Roots	Solutions to a quadratic equation/function $ax^2 + bx + c = 0$	
		The x values where the graph crosses the x axis	
		A quadratic can have 0, 1 or 2 roots	
3.	Quadratic graph	Curved shaped called a parabola	
		A positive x^2 will give a '∪' shape	
		A negative x^2 will give a '∩' shape	
4.	Turning points	The point where a curve turns in the opposite direction	

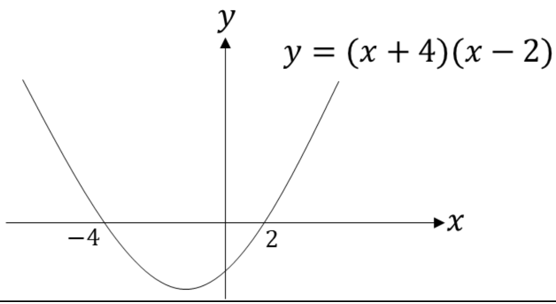
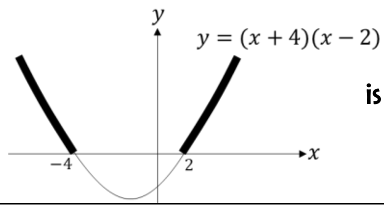
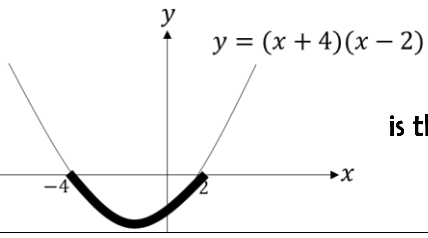
Using the discriminant

5.	Discriminant	The part of the quadratic formula under the square root	$b^2 - 4ac$
6.	$b^2 - 4ac > 0$	Two distinct real roots	
7.	$b^2 - 4ac = 0$	One repeated real root	
8.	$b^2 - 4ac < 0$	No real roots	

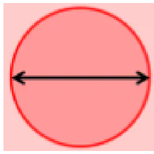
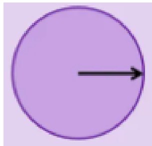
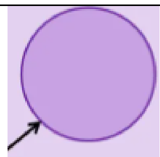
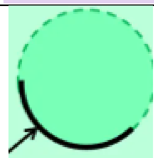
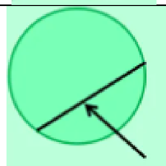
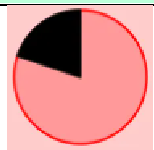
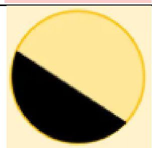
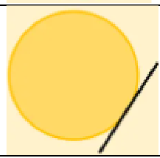
Skletching quadratic graphs

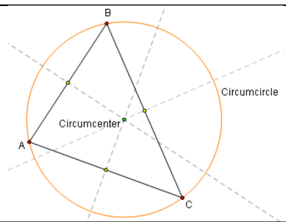
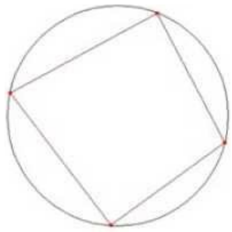
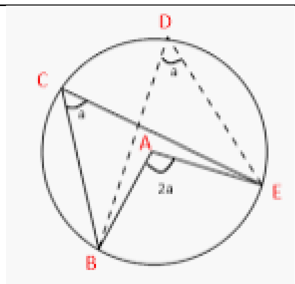
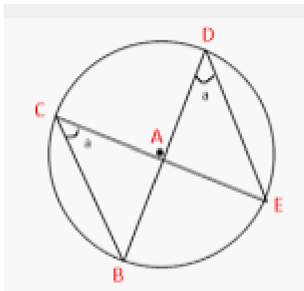
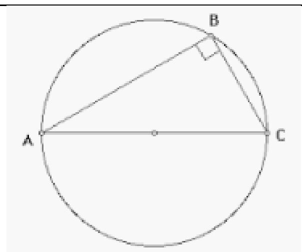
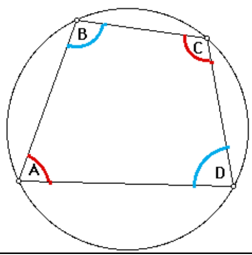
9.	General shape	A positive x^2 will give a '∪' shape A negative x^2 will give a '∩' shape	
	Find the roots	By factorising or using the formula	Equation must be equal to zero
	Find the y intercept	Substitute x =0 zero into the equation	
	Calculate the coordinates of the turning point	Complete the square to get in the form of $\mathrm{f}(x) = a(x+p)^2 + q$	Coordinates of turning point are then $(-p, q)$

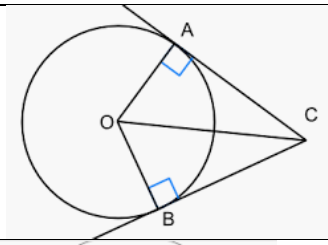
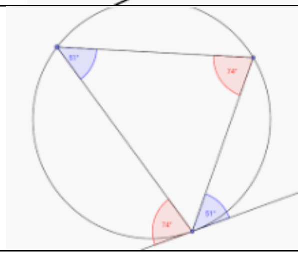
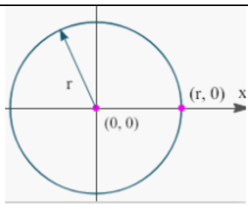
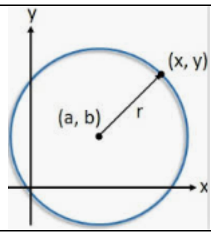
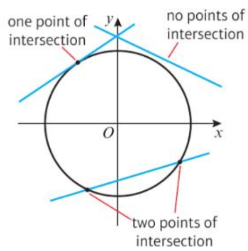
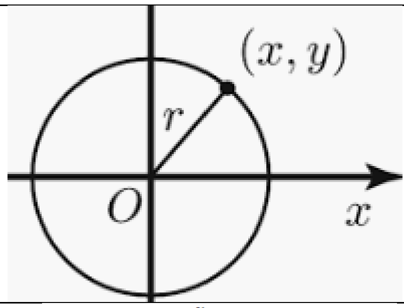
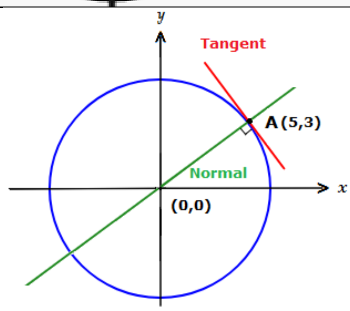
Solving quadratic inequalities

10.	Solve (by factorising or using quadratic formula) $ax^2 + bx + c = 0$	e.g $x^2 - 2x + 8 = 0$ $(x + 4)(x - 2) = 0$ $x = -4 \; or \; x = 2$
11.	Sketch the graph clearings showing the roots and parabola shape	
12.	Check whether your quadratic was greater than or less than zero then highlight parts of the graphs that satisfy this	If $x^2 - 2x + 8 > 0$  Therefore $x < -4$ or $x > 2$ is the solution If $x^2 - 2x + 8 < 0$  Therefore $-4 < x < 2$ is the solution

Circles - definitions and formulae

1.	Diameter	A straight line from edge to edge passing through the centre Double the size of the radius	
2.	Radius	A straight line from the centre to the edge Half the size of the diameter	
3.	Radii	The plural of radius	
4.	Circumference	Distance around the outside of the circle	
5.	Arc	Part of the circumference	
6.	Chord	A line within a circle where each end touches the edge	
7.	Sector	The region created by two radii and an arc	
8.	Segment	The region created by a chord and an arc	
9.	Tangent	A line outside the circle which only touches the circumference at one point	
10.	Semi -circle	Half a full circle	
11.	Line segment	A finite part of a straight line with two distinct endpoints	
12.	Perpendicular bisector	A straight line that is perpendicular to the line L and passes through the midpoint of L	

<table>
<tr><td>13.</td><td>Circumcircle</td><td>A unique circle that passes through all three vertices of a triangle</td><td rowspan="2">
</td></tr>
<tr><td>14.</td><td>Circumcentre</td><td>The centre of a circumcircle, where the perpendicular bisectors of the sides of the triangle intersect</td></tr>
<tr><td>15.</td><td>Cyclic quadrilateral</td><td>A quadrilateral with all four vertices on the circumference of a circle</td><td></td></tr>
<tr><td colspan="4"><h2>Circle Theorems</h2></td></tr>
<tr><td>16.</td><td>Angles at the centre</td><td>Angle at the centre is twice the angle at the circumference</td><td>
</td></tr>
<tr><td>17.</td><td>Angles in the same segment</td><td>Angles at the circumference in the same segment are equal</td><td>
</td></tr>
<tr><td>18.</td><td>Angles in a semi-circle</td><td>Angle in a semi-circle is 90◦</td><td>
</td></tr>
<tr><td>19.</td><td>Cyclic quadrilateral</td><td>Opposite angles of a cyclic quadrilateral add to 180◦</td><td>
</td></tr>
</table>

20.	Tangent to a circle	Angle between a tangent and radius is 90° Two tangents from the same point to a circle are equal in length	
21.	Alternate segment	Angles in the alternate segment are equal	
Circle geometry			
22.	Equation of a circle	With centre $(0,0)$ and radius, r $x^2 + y^2 = r^2$ 	With centre (a, b) and radius, r $(x-a)^2 + (y-b)^2 = r^2$ 
23.	Intersections between circles and lines	• No intersection • Once (where the line touches the circle • Twice (where the line crosses the circle)	
24.	Gradient of a radius to a circle	Gradient (m) of radius to a point$(x, y)$with an equation $x^2 + y^2 = r^2$ is $\frac{y}{x}$	
25.	Gradient of tangent to a circle	Gradient (m) of tangent to a point(x, y) is the negative reciprocal of the gradient of the radius at the same point	

GCSE Mathematics
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Unit 17

Surds

1.	Surd	A number written exactly using square or cube roots	e.g. $\sqrt{5}$ is a surd but $\sqrt{25}$ is not because it has a value of 5
2.	Rationalise	Eliminate a surd	
3.	Multiply	$\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ and $\sqrt{a}\times\sqrt{a}=a$	e.g. $\sqrt{2}\times\sqrt{3}=\sqrt{6}$ and $\sqrt{3}\times\sqrt{3}=3$
4.	Divide	$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$	e.g. $\frac{\sqrt{6}}{\sqrt{2}}=\sqrt{\frac{6}{2}}=\sqrt{3}$
5.	Add and subtract	$\sqrt{a}+\sqrt{b}$ cannot simplify	e.g. $\sqrt{3}+\sqrt{2}=\sqrt{3}+\sqrt{2}$
		But $\sqrt{a}+\sqrt{a}=2\sqrt{a}$	e.g. $5\sqrt{2}-2\sqrt{2}=3\sqrt{2}$
6.	Simplify	$\sqrt{50}=\sqrt{25\times 2}=\sqrt{25}\times\sqrt{2}=5\sqrt{2}$	e.g. $\sqrt{50}+\sqrt{18}=5\sqrt{2}+3\sqrt{2}=8\sqrt{2}$
7.	Rationalise the denominator	Multiply numerator and denominator (use equivalent fractions) by whatever will result in the denominator simplifying to an integer.	e.g. $\frac{1}{\sqrt{7}}=\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{7}{\sqrt{7}}$
			e.g. $\frac{1}{5+\sqrt{2}}=\frac{1}{5+\sqrt{2}}\times\frac{5-\sqrt{2}}{5-\sqrt{2}}=\frac{5-\sqrt{2}}{3}$

Algebraic Fractions

8.	Simplifying	Cancel common factors (factorising if needed)	$\frac{(x-3)(x+2)}{(x+2)(x+5)}=\frac{x-3}{x+5}$
9.	Adding and subtracting	Find a common denominator	$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$
10.	Multiplying	Multiply as with normal fraction	$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
11.	Dividing	Divide as with normal fractions	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

Changing the subject of a formula

12.	Always use inverse operations to isolate the term you have been asked to make the subject		
	If the letter you want as the subject appears twice you will need to factorise		
	Make u the subject: $v = u + at$ $(-\boldsymbol{at})$ $v - at = u$ So $u = v - at$	Make u the subject: $v^2 = u^2 + 2as$ $(-\boldsymbol{2as})$ $v^2 - 2as = u^2$ $(\sqrt{\ })$ $\sqrt{v^2 - 2as} = u$ So $u = \sqrt{v^2 - 2as}$	Make m the subject: $I = mv - mu$ $(\boldsymbol{Factorise})$ $I = m(v - u)$ $(\div (\boldsymbol{v} - \boldsymbol{u}))$ $\frac{I}{v - u} = m$ So $m = \frac{I}{v - u}$

Algebraic proof

13.	Proof	A logical argument fro a mathematical statement
		Use algebra to prove something is true/untrue for all cases
14.	Counter example	Use an example that does not fit the statement to prove the statement is incorrect
Notation to use in proof		
15.	n	Any number
16.	n + 1	Consecutive number
17.	2n	Even number
18.	2n + 2	Consecutive even number to 2n
19.	2n + 1	Odd number
20.	2n + 3	Consecutive odd number to 2n + 1
21.	an	A multiple of a e.g. 3n represents a multiple of 3

Functions

22.	Function	A rule for working out values of y (output) given values of x (input)		
23.	$f(x)$	Function notation read as 'f of x', where x is the input into the function		
24.	Composite functions	$fg(x)$	Evaluate $g(x)$ first then substitute this into $f(x)$	
25.		$gf(x)$	Evaluate $f(x)$ first then substitute this into $g(x)$	
26.	Inverse fuction	$f^{-1}(x)$	Reverses the effect of the original function	$f(x)=3x+2$ $f^{-1}(x)=\frac{x-2}{3}$

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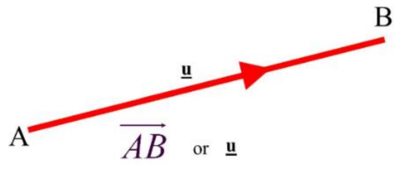
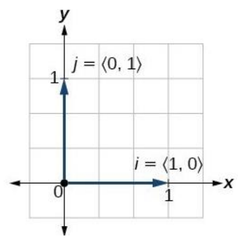
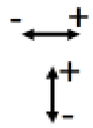
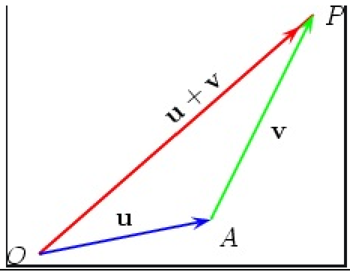
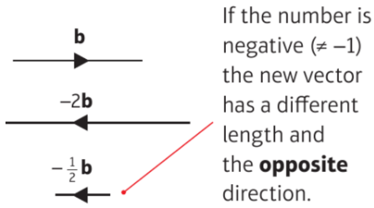
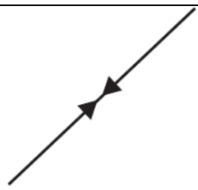
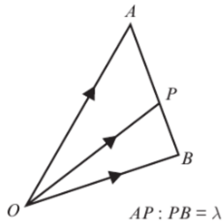
Unit 18

Definitions and processes

1.	Magnitude	Size	Denoted using straight lines on either side of the vector $\|a\|$
2.	Vector	A quantity that has both magnitude and direction	e.g. velocity displacement force
3.	Directed line segment	Can be used to represent a vector	
		Can be written in bold **a**, with underlining $\underline{a}$ or $\overrightarrow{AB}$	
4.	Unit vector	A vector with a magnitude of 1	
		Unit vector in the x direction	
		Unit vector in the y direction	
5.	Column vector	x denotes the horizontal movement	$\begin{pmatrix} x \\ y \end{pmatrix}$ 
		y denotes the vertical movement	
6.	Resultant	The vector sum of two or more vectors	
7.	Displacement	The action of moving something from its place or position	
8.	Scalar	A quantity that has magnitude	e.g. speed is the magnitude of the velocity vector
9.	Colinear	Two vectors that lie on the same line	

10.	Triangle law	$\overrightarrow{OA} + \overrightarrow{AP} = \overrightarrow{OP}$	
11.	Parallel vectors	Any vector parallel to the vector $\boldsymbol{a}$ may be written as $\lambda\boldsymbol{a}$, where λ is a non-zero scalar	
12.	$\begin{pmatrix} p \\ q \end{pmatrix}$	Can also be written as $p\mathbf{i} + q\mathbf{j}$	e.g. $5\mathbf{i} + 2\mathbf{j} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$
13.	Zero vector	$\overrightarrow{OA} + \overrightarrow{AO} = 0$	
14.	Vectors and ratios	If P is A point on AB, dividing AB in the ratio $\lambda : \mu$	$\overrightarrow{OP} = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}\overrightarrow{AB}$ 

GCSE Mathematics
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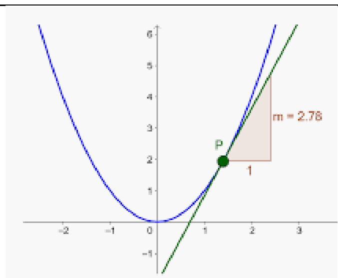
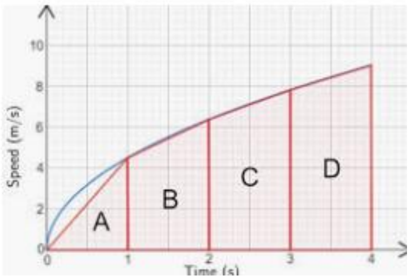
Unit 19

Proportion

1.	Constant of proportionality	Represented by k	
		Its value stays the same	
2.	Direct proportion	Two quantities increase at the same rate	e.g. y is directly proportional to x’ $y \propto x$ $y = kx$
3.	Inverse proportion	One variable increases at a constant rate while the other variable decreases	e.g. ‘y is inversely proportional to x’ $y \propto \frac{1}{x}$ $y = \frac{k}{x}$

Graph transformations

4.	$y = -f(x)$	Reflection in the x axis	y coordinates are multiplied by -1
5.	$y = f(-x)$	Reflection in the y axis	x coordinates are divided by -1
6.	$y = -f(-x)$	Reflection in the x axis and then in the y axis Equivalent to rotation of 180◦ about the origin	y coordinates are multiplied by -1 AND x coordinates are divided by -1
7.	$y = f(x) + a$	Translation by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$	
8.	$y = f(x + a)$	Translation by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$	
9.	$y = af(x)$	Stretch by scale factor a in the vertical direction, parallel to the y axis	y coordinates are multiplied by a
10.	$y = f(ax)$	Stretch by scale factor $\frac{1}{a}$ in the horizontal direction, parallel to the x axis	x coordinates are multied by $\frac{1}{a}$

<table>
<tr><th colspan="4">Rates of change</th></tr>
<tr><td>11.</td><td>Gradient</td><td>The gradient of the tangent to a curve can be used to calcuakte the gradient of a curve at any point</td><td>
</td></tr>
<tr><td rowspan="2">12.</td><td rowspan="2">Area under graph</td><td>The area under the graph represents the product of the units on the y and x axes</td><td>If the graph is a curve then split up into shapes such as trapezia and triangles to find an estimate for the area</td></tr>
<tr><td>e.g. for a velocity time graph the area represents the distance travelled</td><td>
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